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## THESIS

ANALYSIS OF THE COMMAND AND CONTROL  
NETWORK MODEL AND LINKAGE MECHANISM  
WITH FORCE EVALUATION MODELS

by

Steven E. Strukel

September 1992

Thesis Advisor:

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Analysis of the Command and Control Network Model  
and Linkage Mechanism with Force Evaluation Models

by

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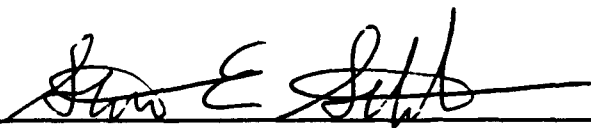
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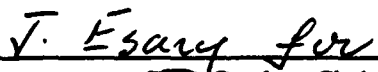
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## ABSTRACT

Newly emerging functional area models designed to simulate the activities of individual battlefield operating systems are generating challenging new validation issues for Army analysts. From database to output, these new models require testing against real system performance to insure no significant disparities exist. The Command and Control Network (C2NET) model is a prototype Command and Control Functional Area Model (C2FAM) and exemplifies this validation challenge. This paper examines the C2NET database, input distributions, and linkage mechanism with a force evaluation model as part of C2NET's continuing validation effort. The nonstationary Poisson process is examined and used to develop hypotheses about input distributions linking the evaluation model's tactical scenario with C2NET's input parameters and existing database. Fuzzy set theory is then examined with applications for using C2NET output as input for an evaluation model. Areas for further research are discussed.

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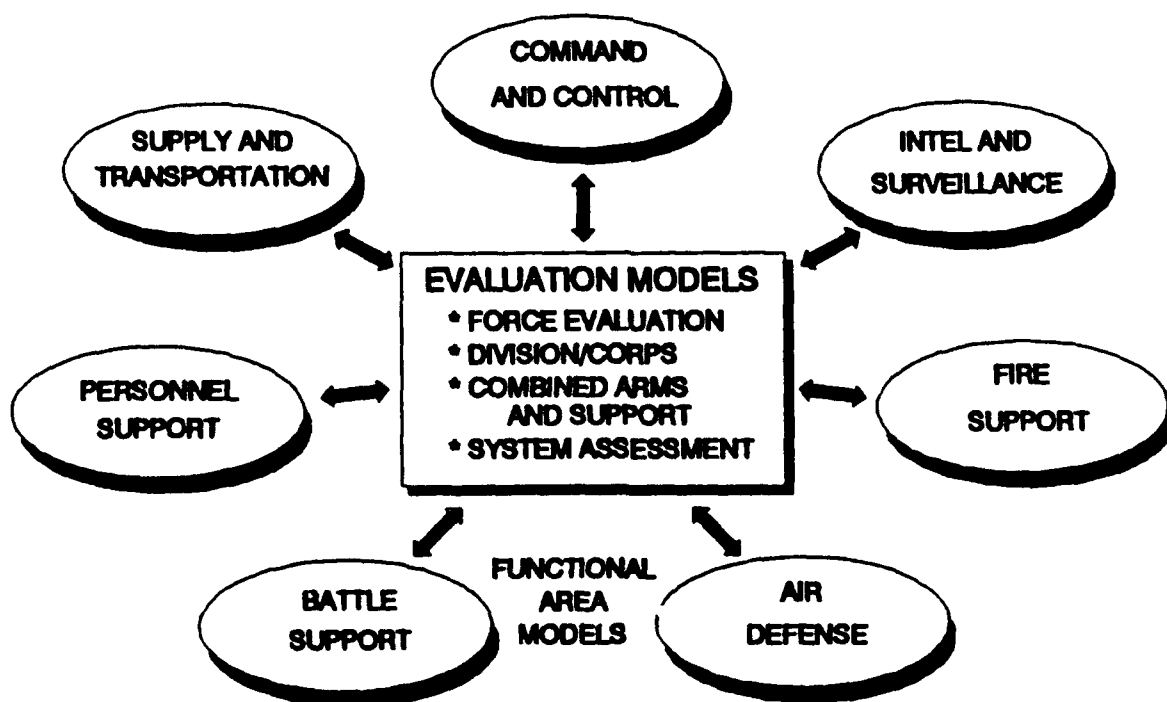
## I. INTRODUCTION

Now if the estimates made...before hostilities indicate victory it is because calculations show one's strength to be superior to that of his enemy; if they indicate defeat, it is because calculations show that one is inferior. With many calculations, one can win; with few one cannot. How much less chance of victory has one who makes none at all! By this means I examine the situation and the outcome will be clearly apparent. (Sun Tzu, 1963, p. 71)

The requirement for valid models to forecast and evaluate military capabilities is certainly no less today than it was for Sun Tzu in 500 B.C.. From force sizing to weapon procurement strategies, the Army of the 1990's is dependent on land combat models for developing a viable force structure for the future under increasingly tight fiscal constraints. The family of land combat models (Figure 1) currently in use and under development is characterized by a hierarchy of evaluation models supported by functional area models.

As implied in their titles, evaluation models represent combat operations over the range of force structure, from individual weapon systems to theater operations. The functional area models typically abstract the consequences of combat operations and concentrate on the activities of a single battlefield operating system. Output from a functional area model can stand alone using acceptable measures of performance (for example, when comparing different systems

with similar functions). Ideally, however, the output from a functional area model is used as input for an evaluation model. Two immediate and obvious benefits result from this linkage; first, the evaluation model better approximates the "reality" of combat, and second, a stronger comparison is available for analyzing the results of changes made in a functional area system through measures of force effectiveness (MOFE). For example, a measure of performance (MOP) for a given system might be "processing time" where System A demonstrates a 10% improvement over System B. A stronger comparison is made when run through an evaluation model, with an MOFE of Red/Blue Casualties and System A shows a 10% increase in the loss exchange ratio over System B.



**Figure 1. Family of Land Combat Models**





These validation issues are in two basic categories; model input and model output validation. Model input encompasses the model database, distributions, and linkage from the evaluation model into C2NET through abstraction of the tactical scenario. Model output considerations include selection of appropriate C2 performance parameters (producible by C2NET and usable by the evaluation model to approximate C2 effects on force effectiveness) and development of a methodology to obtain these parameters (the model post-processor).

C2NET in its current embryonic state uses subject matter expert (SME) point estimates to approximate the minimum, maximum, and mode for key C2 task frequencies and task durations. While a valid technique in model development, it fails to simulate the underlying theoretical distributions (if present) and does not provide for linkage between the tactical scenario and task generation within C2NET. Analysis of available empirical data yields valuable insights about the nature of the distributions in question, allowing us to develop and test hypotheses that better approximate the behavior of C2 systems. The central concept behind these hypotheses is the nonstationary (or nonhomogeneous) Poisson process. This process allows for the possibility that arrival rates may vary as a function of time, an essential attribute when linking the tactical scenario (operational tempo) into C2NET. We expect the arrival rate for most C2 tasks to

increase as the operational tempo (OPTEMPO) increases (for example, we should observe a low number of spot reports when a unit is not in contact, with a dramatic increase in spot reporting during combat). Examining empirical data observed during periods of "high," "medium," and "low" operational tempo will allow us to identify underlying theoretical distributions linked to the tactical scenario and finally test these distributions using parameters from the existing database. The end result of this process is a C2NET model sensitive to the activities within the evaluation model and better approximating real world performance of command and control systems.

Selection of reasonable and useful C2 performance parameters for inclusion into the evaluation model is a harder problem. How do we take the output from the highly stochastic C2NET and develop input for the totally deterministic evaluation model? The current solution is to manually calculate single values for each echelon in maneuver delays (required time spent in "dwell points" for moving forces, approximating the delays observed in C2NET for the orders process) and artillery fire delays (approximating the lag time observed in the C2NET model in response to requests for fire support). Clearly command and control effects are not just delay times in maneuver and artillery fire, but approximating other, less tangible effects, has thus far been resistant to solution using conventional modelling techniques.

For example, how do we efficiently approximate the command and control measures observed in C2NET for "Artillery Effectiveness" such that the results are usable in an evaluation model? Simply defining "Artillery Effectiveness" in this context is a difficult task; what factors should be included in our definition and at what weight? A solution to the C2NET output dilemma can be found in the theory of fuzzy sets. Briefly, fuzzy set theory allows for "shades" of effectiveness beyond the crisp (binary) yes or no answer to the question of C2 performance in conventional modelling. Applying fuzzy set theory to our "Artillery Effectiveness" example, we may allow a spectrum from "poor" to "excellent" and use C2NET output to classify effectiveness for each unit by time period. In the evaluation model, these variables are then used to determine not only delay factors, but adjustments to fire power effectiveness (through reductions in attrition coefficients). This proposal and its underlying concepts receive a more detailed treatment in Chapter III, but by way of introduction, we see it is possible to approximate abstract concepts efficiently in an evaluation model.

Finally, as Payne writes, "Another aspect of the review processes necessary to the validation of the family of models, or its individual members, is the recognition that no finding of validity through review is final or permanent." (1989, p.279). This must be especially true for a new model (C2NET) as a member of a new family of models (C2FAM). Additional

hypothesis testing will require a significant effort in data collection through observation of C2 systems during training or operational deployments. Acceptance of fuzzy sets as C2 performance measures will generate additional requirements for SME input and testing. Implementation of changes in model input parameters will entail recoding significant portions of C2NET and subsequent sensitivity analysis. Where appropriate, methodologies to implement future work are identified and discussed in Chapter IV.

At a minimum, the analysis of C2NET input and output parameters will provide a guide to future validation efforts and highlight the current potentials for significantly increasing model fidelity.

## II. COMMAND AND CONTROL NETWORK INPUT VALIDATION

*Validation.* Checking that the simulation model, correctly implemented, is a sufficiently close approximation to reality for the intended application. As already indicated, no recipe exists for doing this. Due to approximations made in the model, we know in advance that the model and the real system do not have identical output distributions; thus, statistical tests of model validity have limited use. The real question is the practical significance of any disparities. (Bratley, Fox, and Schrage, 1987, pg. 8)

The first step in validating the C2NET model is analysis of the model's existing database and input distributions. What approximations have been made in the model's input parameters? Next, using empirical data and knowledge of the system, we identify improvements that will reduce disparities between the model and real system input. For C2NET, this step involves identification of the underlying theoretical input distributions with appropriate parameters as well as a methodology to link C2NET with the tactical scenario of the evaluation model. Finally, we must test our new construct against empirical data of real world system performance and analyze the results for robustness (is our new model resistant to inaccuracies in the database?). The end result of this validation of C2NET input parameters is a more robust model better approximating real C2 systems with a strong linkage mechanism from the evaluation model, allowing C2NET to emulate events in the tactical scenario.

## A. ANALYSIS OF THE CURRENT C2NET INPUT PARAMETERS

The C2NET database was developed by the C2 Analysis Cell (Combined Arms Center [CAC], Fort Leavenworth). The database consists of frequencies<sup>1</sup>, completion times, and the personnel required to complete key C2 tasks. An argument for the heuristic approach used to develop the database's estimates is proposed by Law and Kelton:

In some simulation studies it may not be possible to collect data on the random variables of interest,.... For example, if the system being studied does not currently exist in some form, collecting data from the system is obviously not possible. This difficulty can also occur for existing systems, if the number of required probability distributions is large and the time available for the simulation study prohibits the necessary data collection and analysis.

Let us assume that the random quantity of interest is a continuous random variable  $X$ . It will also be useful to think of this random variable as being the time to perform some task, e.g., the time required to repair a piece of equipment when it fails. The first step...is to identify an interval  $[a,b]$  (where  $a$  and  $b$  are real numbers such that  $a < b$ ) in which it is felt that  $X$  will lie with probability close to 1; that is,  $P(X < a \text{ or } X > b) \approx 0$ . In order to obtain subjective estimates of  $a$  and  $b$ , "experts" are asked for their most optimistic and pessimistic estimates, respectively, of the time to perform the task.

In the triangular approach, the experts are also asked for their subjective estimate of the most likely time to perform the task. This most likely value  $c$  is the mode of the distribution of  $X$ . Given  $a$ ,  $b$ , and  $c$ , the random variable  $X$  is then considered to have a triangular distribution on the interval  $[a,b]$  with mode  $c$ . (Law and Kelton, 1991, pg. 403)

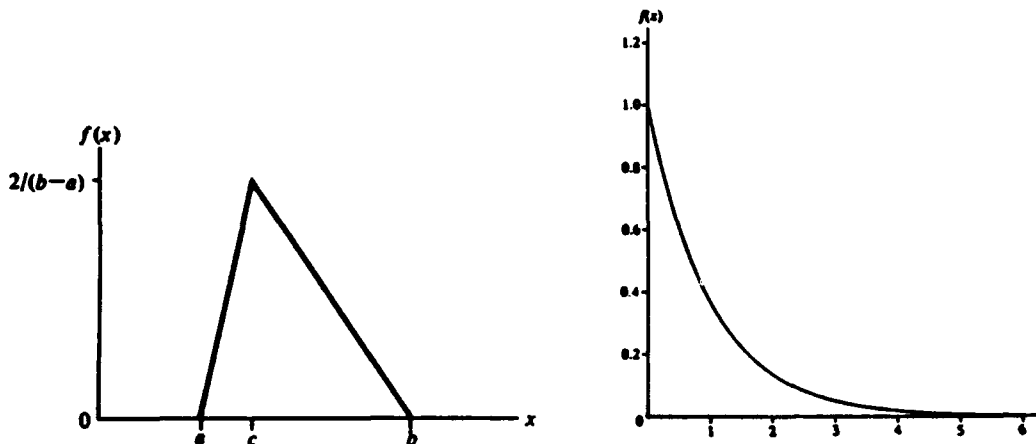
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<sup>1</sup> In C2NET, estimates for task frequency have been converted to estimates of interarrival times, e.g., a task with MIN/MODE/MAX estimates of task frequency as 3/5/10 would have MIN/MODE/MAX interarrival estimates of 6/12/20.

Two studies (the C2 Responsiveness Analysis and the Division Command Post Study) have been completed with the triangular distribution database driving task generation and completion within C2NET. These studies successfully demonstrated C2NET's ability to approximate command and control effects in an evaluation model. However, they also highlighted the necessity of a link between the tactical scenario and C2NET. Without this linkage, at best C2NET can only provide an aggregate measure of C2 effectiveness over the life of the scenario. The resulting loss of information (due to aggregation) is a potential source of significant error when comparing command and control alternatives.

Another source of error encountered using the database in its current form is the approximation made with the triangular distribution. A task with arrivals behaving according to a Poisson process in the real system (interarrival times exponentially distributed) and modeled with a triangular distribution will not give us an accurate picture of real system performance. Compounded over many tasks, the disparities in modeled and real world system performance may lead us to inappropriate conclusions when comparing C2 systems using C2NET. A graph of a triangular distribution contrasted with an exponential distribution plot is at Figure 3, and highlights the disparity between these distributions.





**Figure 3. Triangular and Exponential Distributions**

## **B. SELECTION OF APPROPRIATE INPUT DISTRIBUTIONS**

The first step in selecting a particular input distribution is to decide what general families--e.g., exponential, normal, or Poisson--appear to be appropriate on the basis of their shapes, without worrying (yet) about the specific parameter values for these families.

In some situations, use can be made of *prior knowledge* about a certain random variable's role in a system to select a modeling distribution or at least rule out some distributions; this is done on theoretical grounds and does not require any data at all. For example, if we feel that customers arrive to a service facility one at a time, at a constant rate, and so that the numbers of customers arriving in disjoint time intervals are independent, there are theoretical reasons...for postulating that the interarrival times are IID exponential random variables. (Law and Kelton, 1991, pp. 356-7)

## 1. Nature and Role of C2 Tasks

Prior to identifying the general families of distributions appropriate for modeling C2 task arrival and completion times, we need to investigate the nature and role of these tasks in the C2 system. In general, each key C2 task can be classified into one of three categories. These categories are defined by their roles in the C2 system and corresponding attributes. The subjective categories are "dynamic," "periodic," and "interactive." Common to all tasks, regardless of category, is the fact that tasks arrive one at a time. For example, even with advances made in multi-channel communication systems we still expect to see only one spot report or request for fire arrive at any moment in a real C2 system. A list of key tasks identified by category and battlefield operating system (BOS) is provided in Appendix A.

"Dynamic" refers to tasks which model activities with arrival rates dependent on the tactical scenario. For example, Process Intelligence Spot Report (PIS) is a dynamic task generated by observation of enemy forces or activity. In this example, a spot report is submitted as a result of newly observed enemy activity and is not dependent on previous enemy activity. As more enemy forces come into contact with friendly units, additional spot reports are generated. We conclude that the number of spot reports generated in any given time period is dependent on the level of enemy activity

and independent of any previously generated spot reports. In general we expect that for any dynamic task we will see arrivals one at a time, at a rate varying with the OPTEMPO of the appropriate BOS, and with the number of arrivals in disjoint intervals independent.

Periodic tasks model activities generated both as a result of events occurring in the tactical context and according to standard operating procedures (SOP). The Update Status of Friendly Forces (UFF) task is an example of a periodic task. In the real system, updates of this type are required on a periodic basis (interval determined by SOP) and when significant changes to the unit status occur. The causes of these changes are dependent on the level of activity in the tactical scenario. Enemy contact will result in losses of combat power, expenditure of supplies, and changes in force disposition, all of which require updates to the unit status in addition to those required by SOP. At high levels of OPTEMPO, we would expect these tasks to arrive in a manner similar to dynamic tasks, as constant changes in unit status generate the update task and few, if any periodic reports are necessary. At low levels of OPTEMPO, we observe the opposite with very few changes occurring in unit status between required periodic situation reports.

Arrivals of periodic tasks at low levels of OPTEMPO should occur with a nearly batch process behavior, reflecting subordinate units submitting situation reports at or about a time specified by SOP--e.g., the SOP might dictate subordinate units to submit a situation report on the hour, plus or minus ten minutes. Outside of this reporting window we should observe few if any arrivals, until the next periodic reports are due or battlefield activity results in changes to unit status.

Medium levels of OPTEMPO can occur when only part of the force is in contact. We should observe periodic reports arriving from units in reserve or in quiet sectors and tasks arriving according to a more dynamic process from units in contact (rapidly changing status). Modeling periodic task arrivals presents a significant challenge, for while we expect to see arrivals one at a time and at a rate varying with OPTEMPO, we must also account for variations in the shape of the arrival distribution based on OPTEMPO.

Interactive tasks are those with task generation dependent solely on activities within the C2 framework. Prepare Fragmentary Order (PREP FRAGO) is an example of an interactive task. Frag orders are prepared in response to a higher headquarters frag order or as a result of critical events in the tactical scenario (within C2NET, each Process Critical Sitrep task has a chance of generating a PREP FRAGO task). Since no independent arrival process is required for

interactive tasks, validating the model behavior of dynamic and periodic tasks will satisfy the arrival process input distribution validation issue for C2NET.

Service times in the real system do not appear to depend on task category or the OPTEMPO of the tactical scenario. Once a task arrives at a server (the task may or may not have waited in a queue for a server), we expect the server to work on that task until completed (task served one at a time) with the service time of any given type of key C2 task independent of previous service times and identically distributed. For example, a spot report of a certain length and complexity may require one minute to process where this processing time should not be affected by the presence (or absence) of additional spot reports in the queue, nor by the time to process previously served spot reports.

## **2. Examination of Empirical Data Sources**

Sargent (1980) points out that validation should include validation of the data used to develop the conceptual model, and the data used for testing. It is usually time-consuming and costly to obtain accurate and appropriate data, and initial attempts to validate a model may fail if insufficient effort has been applied to this problem. A model that merely accommodates known facts is just an *ad hoc* fit. A worthwhile model must generate predictions that are then corroborated by observations of, or experiments with, the real system. If this process leads to really new insight, then the model takes on the status of a theory. (Bratley, Fox, and Schrage, 1987, pp. 8-9)

Before selection of input distributions, it is necessary to examine what data will be used to develop and test our model. Although much has been done by Army analytical agencies in collecting observations of C2 systems in action, most of this work fails Sargent's criteria as "accurate and appropriate data" when applied to C2NET validation. At a minimum, empirical data to support our effort should be current, targeted to observation of key tasks represented in C2NET, and with a sufficient sample size to provide a clear picture of potential underlying theoretical distributions (also insuring statistical significance during eventual hypothesis testing). Data collected as a result of two different studies meet these criteria and are used in our C2NET validation effort.

The first data source is from a 1990 study of division level C2 by the Army Research Institute (ARI). Although the collection effort was for ARI internal use and no information was gathered on task processing times, the data regarding arrival times are usable for analyzing at least one task. Our primary source of empirical data for both C2 task arrival and duration is from a C2 Analysis Cell study of SANTA FE V. SANTA FE V was a brigade level command post exercise (CPX) conducted 22-24 February, 1991 at Fort Leavenworth, Kansas.

The data collected from these studies are sufficient to examine the following tasks:

- Process Intelligence Spot Report (PIS, Dynamic, Division Level, Arrival Process Only)
- Process Critical Situation Report (PCS, Dynamic, Brigade Level, Task Arrival and Duration Data)
- Update Status of Friendly Forces (UFF, Periodic, Brigade Level, Task Arrival and Duration Data)

The small number of tasks represented in our test data (three out of a possible 105) will preclude hypothesis testing on any significant scale regarding the entire database. However, the available test data are sufficient to allow examination of our statements about the behavior of C2 tasks through comparison of empirical distributions with the theoretical distributions suggested from our knowledge of C2 task behavior. Additionally, hypothesis testing using the empirical data will generate valuable information about our conceptual model--e.g., p-values and confidence intervals allow us to draw conclusions about model resistance to errors in parameter estimation and quality of fit. These conclusions in turn will indicate potential refinements in our concepts of the behavior of C2 tasks, with subsequent alterations to our model resulting in predictions consistent with Bratley's criteria for a "worthwhile model."

### 3. C2 Task Arrival Models

One question that may arise when choosing probability distributions for the model is whether to use frequency distributions of historical data or to seek the theoretical probability distribution which best fits these data. The latter alternative usually is preferable because it would seem to come closer to predicting expected future performance rather than reproducing the idiosyncracies of a certain period of the past. (Law and Kelton, 1991, pp. 647-8)

Given our previous examination of the behavior of the C2 task arrival processes and usable test data, we are now prepared to search for the appropriate theoretical distributions as suggested by Law and Kelton.

#### *a. Dynamic Task Arrival Distributions*

The behavior of dynamic tasks suggests that if we let the random variable  $X$  represent the arrival times for one of these tasks, then  $X$  is independently and exponentially (but not identically) distributed with arrival rates determined by the task's BOS OPTEMPO (arrival rates are piecewise constant given OPTEMPO). This type of random variable is said to be generated by a nonstationary (or nonhomogeneous) Poisson process. We turn again to Law and Kelton for an explanation of the attributes for this process:

Let  $\lambda(t)$  be the arrival rate of customers to some system at time  $t$ . If customers arrive at the system in accordance with a Poisson process with rate  $\lambda$ , then  $\lambda(t) = \lambda$  for all  $t \geq 0$ . However, for many real-world systems,  $\lambda(t)$  is actually a function of  $t$ . For example, the arrival rate of customers to a fast-food restaurant will be larger during the noon rush hour than in the middle of the



afternoon. Also, traffic on a freeway will be heavier during the morning and evening rush hours. If the arrival rate  $\lambda(t)$  does in fact change with time, then the interarrival times  $A_1, A_2, \dots$  are not identically distributed; thus, it is not appropriate to fit a single probability distribution to the  $A_i$ 's...

The stochastic process  $\{N(t), t \geq 0\}$  is said to be a *nonstationary Poisson process* if:

1. Customers arrive one at a time.
2.  $N(t + s) - N(t)$  is independent of  $\{N(u), 0 \leq u \leq t\}$ .

Thus, for a nonstationary Poisson process, customers must still arrive one at a time, and the numbers of arrivals in disjoint intervals are independent, but now the arrival rate  $\lambda(t)$  is allowed to be a function of time. (Law and Kelton, 1991, pp. 406-7)

The arrival rate as a function of time is the key to linking the force evaluation model's tactical scenario with C2NET. The OPTEMPO of each BOS is driven by the tactical scenario. Quantifying this OPTEMPO by BOS over the life of the scenario into "OPTEMPO Timelines" and inputting the timelines into C2NET as governors over task arrival rates will satisfy our linkage requirement.

The importance of the nonhomogeneous Poisson process resides in the fact that we no longer require the condition of stationary increments. Thus we now allow for the possibility that events may be more likely to occur during certain times during the day than during other times. (Ross, 1989, pg. 235)

The approximation made in categorizing the continuous distribution of OPTEMPO into three discrete levels as High, Medium, and Low is made on the following basis:

- This is a *sufficient* level of detail. Examination of the test data indicates three levels of OPTEMPO will avoid significant disparity between our approximation and the real system.
- This is a *practical* level of detail. OPTEMPO timelines must be created for each Brigade and higher unit, for each battlefield operating system, for every scenario C2NET is to approximate. Additional levels of detail would increase the classification work significantly.
- This is a *useful* level of detail. The C2NET database consists of three estimates for MIN/MODE/MAX interarrival times. Our hypothesis for task arrival processes will link this database with parameter estimates at the three levels of OPTEMPO.

The first step in analyzing our test data for conformity with the nonstationary Poisson process is to create the OPTEMPO timelines for the applicable BOS. For the SANTA FE test data, we need to construct Intelligence and Electronic Warfare (IEW) and Maneuver (MVR) BOS timelines (for the PCS and UFF tasks respectively). The ARI data require an OPTEMPO timeline for the IEW BOS (the PIS task). In developing the timelines for these BOS, no information was available regarding events within the scenarios and the timelines in Figures 4 and 5 reflect subjective estimates of OPTEMPO.<sup>2</sup>

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<sup>2</sup> A methodology is discussed in Chapter IV as a guide in developing OPTEMPO timelines from the force evaluation model scenario.

These estimates are based on the test data and background information on the studies. The timeline estimation procedure may have introduced error into our study through bias and incorrect identification of timeline OPTEMPO levels. Attempts to reduce error through independent analysis of the OPTEMPO timeline will not totally eliminate these types of error and we must remain aware of this problem during our examination of the input process as a potential source of disparity.

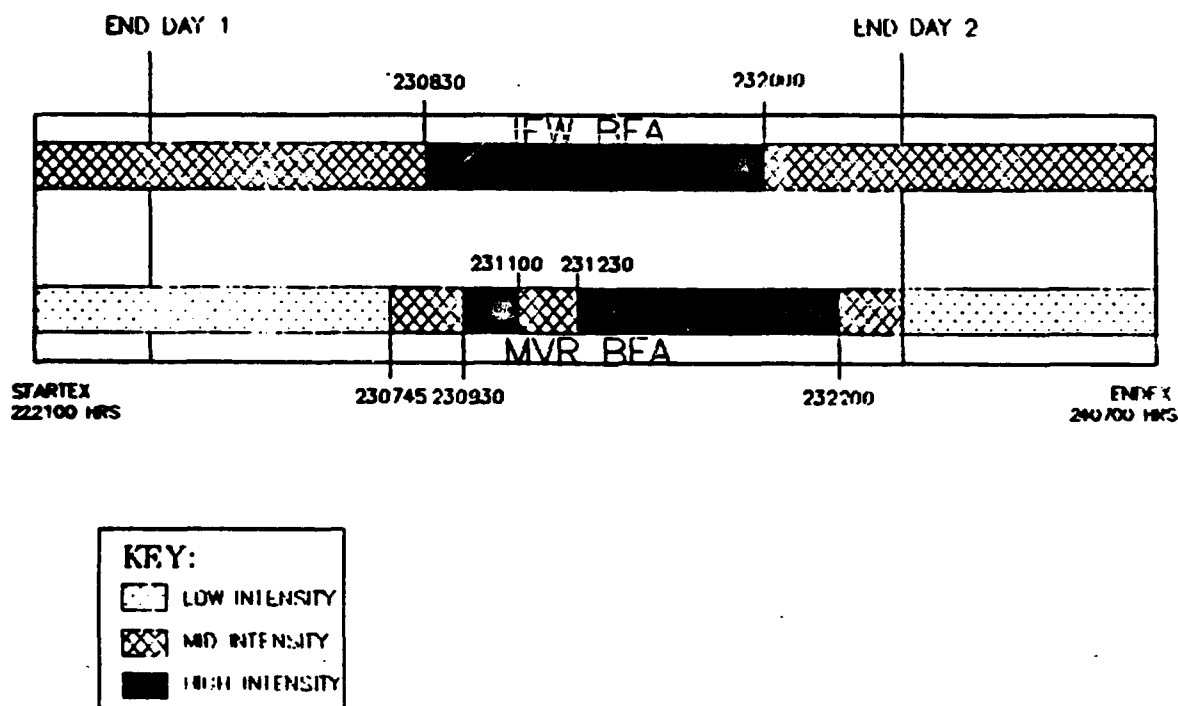
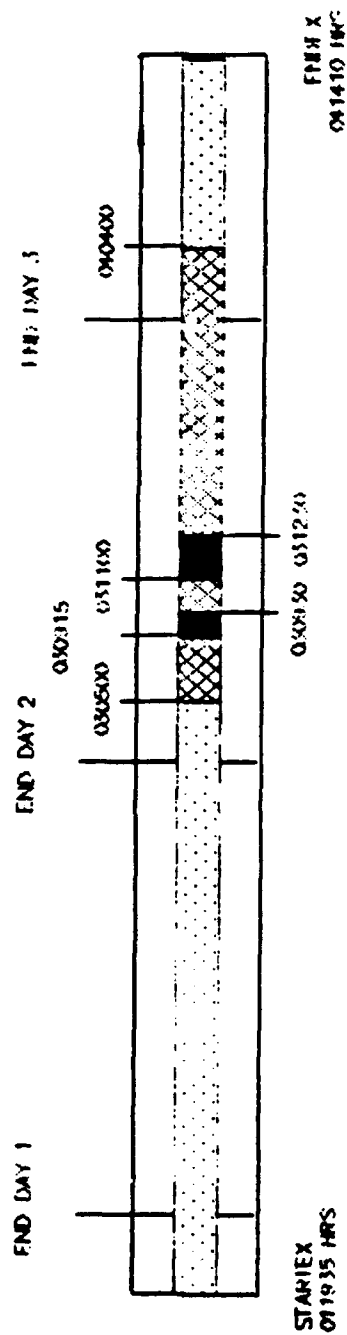


Figure 4. SANTA FE Study OPTEMPO Timelines

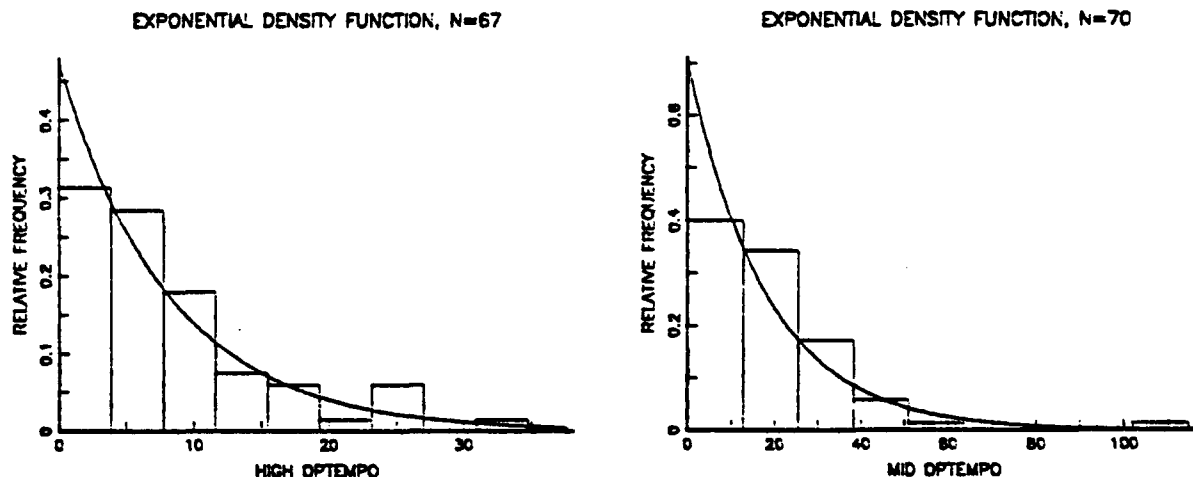


NOTE: CHANGES IN INTENSITY DETERMINED  
THROUGH ANALYSIS OF RAW DATA. NO SCENARIO  
INFORMATION IS AVAILABLE.

KEY:	
[ ]	LOW INTENSITY
[X]	MID INTENSITY
[■]	HIGH INTENSITY

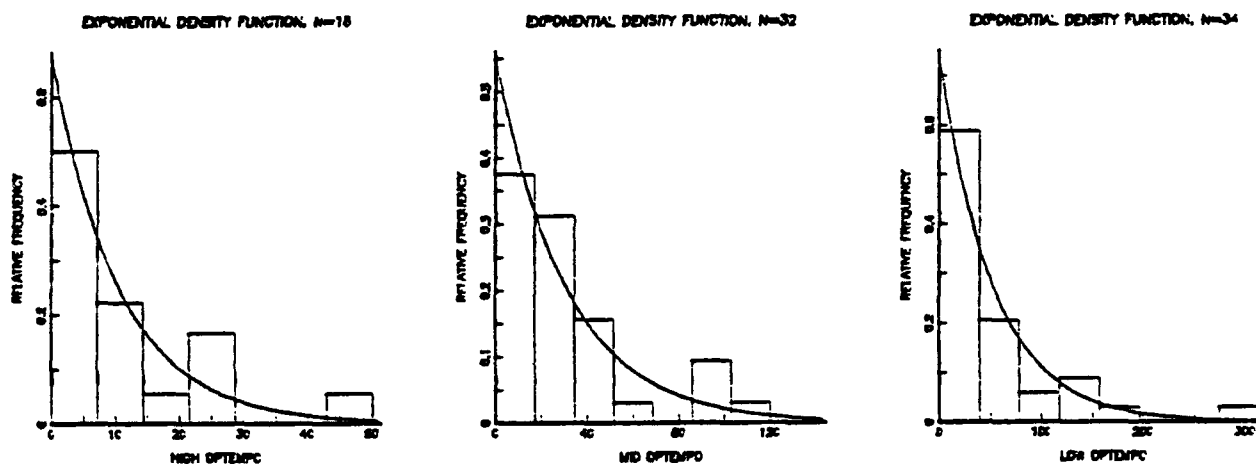
Figure 5. ARI Study OPTEMPO Timeline

The next step suggested by our analysis of the arrival behavior of dynamic tasks is to directly compare the test data against the exponential distribution. Using the GRAFSTAT analytical tool, we will jointly plot the histogram of the test data interarrival times with the exponential density function. For these comparisons, we allow GRAFSTAT to select the input parameter yielding the best exponential fit to our test data. Figure 6 shows the plots for the PCS task from the SANTA FE study. Plots for the PIS task are in Figure 7. Additional types of plots (CDF and Probability-Probability [P-P] plots) are at Appendix B comparing test data with the parameter free exponential distribution.



**Figure 6. PCS Histogram with Exponential Distribution**

Visual inspection of these plots tends to confirm our analysis of the arrival process for the PCS dynamic task. Both at medium and high OPTEMPO, the test data appear reasonably well fit by the exponential distribution. At this stage, this is sufficient to continue our hypothesis development process.



**Figure 7. PIS Histogram with Exponential Distribution**

As we observed with the test data for the PCS task, examination of the plots for the PCS dynamic task agrees with the suggestion that the dynamic task arrival process is nonstationary Poisson. Each plot, from high to low OPTEMPO, appears to support the argument of an underlying exponential distribution, with rate varying by OPTEMPO.

The results of the comparisons made using test data on dynamic tasks against the exponential distribution are sufficiently encouraging to continue with formal hypothesis construction and testing. First however we must complete the selection of input distributions for arrival models (periodic tasks) and task duration models.

***b. Periodic Task Arrival Distributions***

Unlike the behavior of dynamic tasks, where arrivals were uniquely identified with the nonstationary Poisson process, no ready explanation exists for the arrival process of periodic tasks. We do know from the behavior of arrivals that the exponential should be the underlying theoretical distribution for periodic tasks during periods of high OPTEMPO. In selecting the underlying distributions for periods of medium and low OPTEMPO, we make the initial assumption that these distributions are of the same type, with changes in shape and scale to approximate the increasingly periodic nature of these tasks.<sup>3</sup> There are two obvious candidate distributions that include the exponential (given the correct shape parameter,  $\alpha=1$ ). These distributions are the Gamma and the Weibull. With this choice, we will

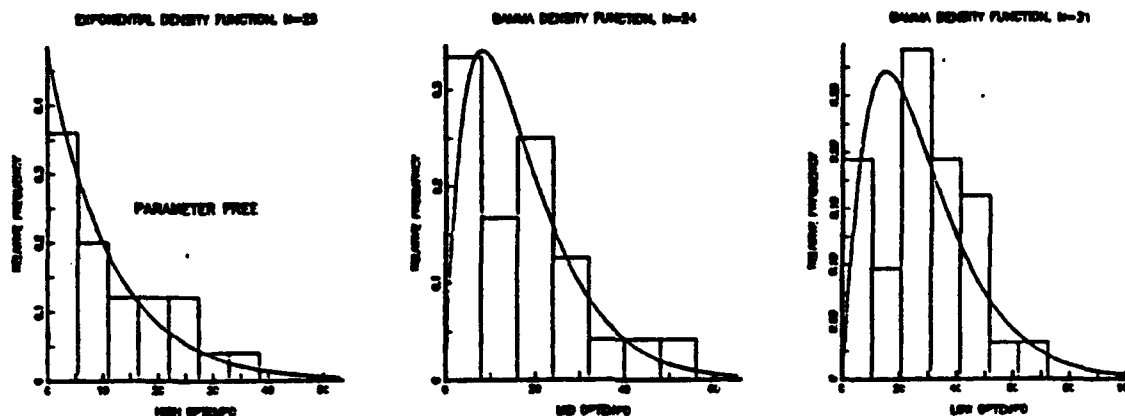
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<sup>3</sup> In C2NET, we will not attempt to simulate the actual periodic nature of these tasks, only the workload generated over time. The resolution of interface between C2NET and the force evaluation model does not justify the degree of detail required to deliberately simulate the periodic lulls and peaks. In effect, we are modeling the steady-state arrival behavior for periodic tasks in a terminating model.

initially select the Gamma distribution for analysis with the option of returning to the Weibull if the Gamma proves unsatisfactory. This selection is based on robustness--the Gamma distribution is more resistant to error in the shape parameter.

From our analysis of the behavior of the nature of the arrival process for periodic tasks, we know to set  $\alpha=1$  during periods of high OPTEMPO. We need to estimate the shape parameters for periods of medium and low OPTEMPO. Previous analysis suggests that the shape parameter for medium OPTEMPO should be less than that for low OPTEMPO. Initially, we will estimate  $\alpha=3$  at periods of low OPTEMPO and  $\alpha=2$  for medium OPTEMPO while allowing the scale parameter to be free (GRAFSTAT selecting  $\beta$  to best fit the test data). The fits obtained proved unsatisfactory with poor behavior in the tails and local modes of the test data (see Figures 23-26 in Appendix B). At the next estimated values of  $\alpha=2$  for low OPTEMPO and  $\alpha=1.5$  for medium OPTEMPO satisfactory fits were obtained. Graphic results of fitting the UFF task interarrival time test data (partitioned according to the maneuver timeline in Figure 4) against the Gamma distribution (with  $\alpha=1/1.5/2$  for HI/MID/LOW OPTEMPO and free scale parameters) are shown in Figure 8.





**Figure 8. UFF Histogram with Gamma Distribution**

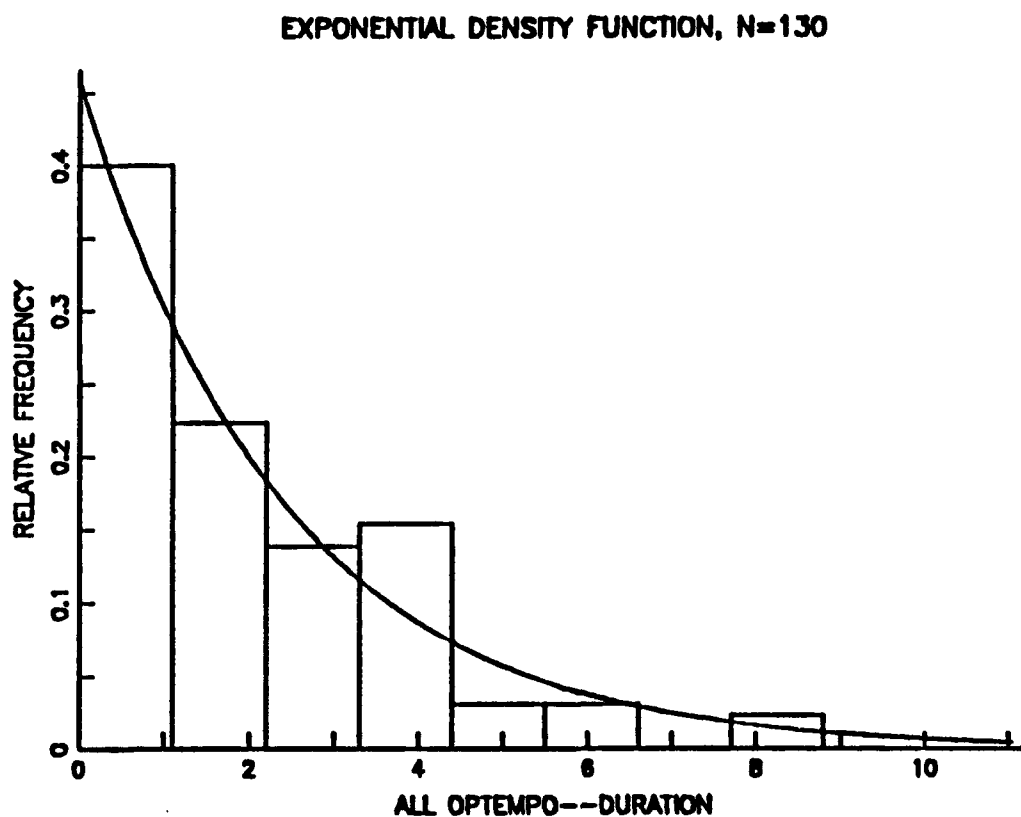
Visual examination of the plots reveals a fairly good fit of the test data by the Gamma distribution with shape parameter estimates as discussed. As expected for the medium and low levels of OPTEMPO there are two local modes formed by the nature of periodic reporting. Selection of the Gamma distribution approximates these local modes into a singular mode, which is satisfactory given our interest in simulating workload (see Note #3, page 25). It may be of interest to future analysts to accurately model the presence of local modes using a composite or piecewise function. However, for purposes of this paper, the Gamma distribution with shape based on OPTEMPO appears to sufficiently approximate the arrival process for periodic tasks.

#### 4. C2 Task Duration Model

In C2NET, each task performed by the system requires a number of personnel over a completion period. The completion period (or duration) is determined by a triangular distribution with parameters estimated by subject matter experts. As for the arrival models, identification of the underlying theoretical distribution for task duration will reduce the disparity between simulation and the real world behavior. The nature of C2 task service times suggests that we examine the exponential as the underlying theoretical distribution for task duration.

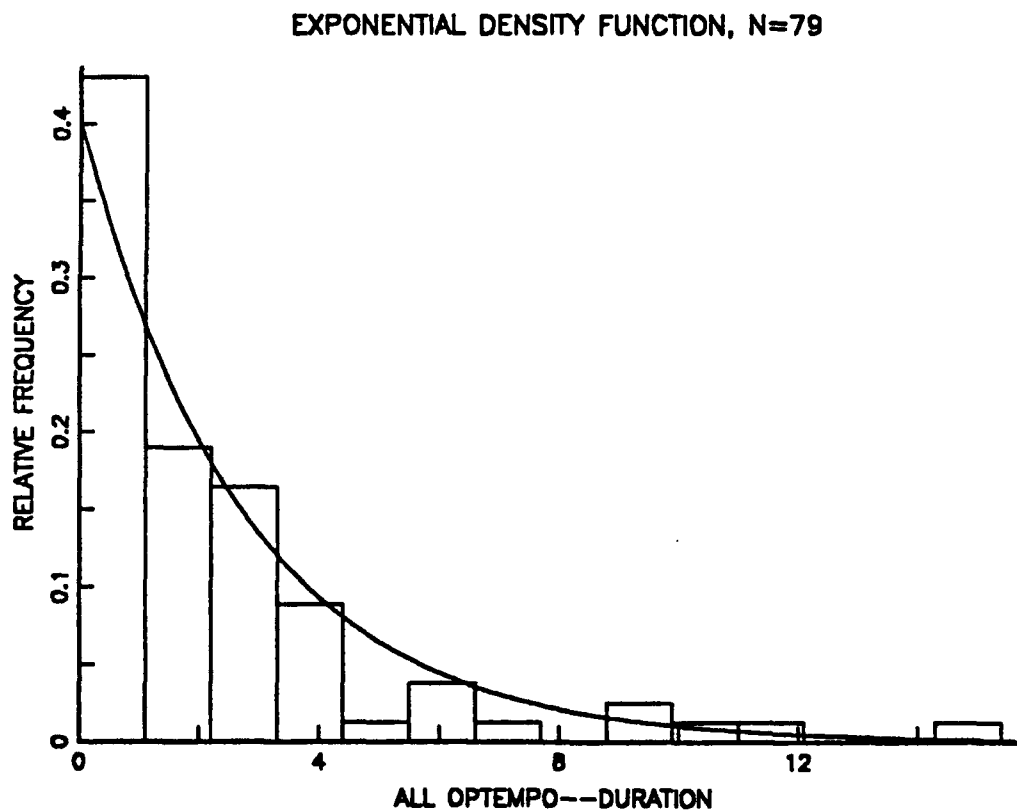
To test the exponential distribution we have observations made of the PCS and UFF tasks during the SANTA FE CPX. There are two problems related to the data collection methodology that will impact on our analysis. The first problem is with the discrete measurement of duration. The time required to complete a task in this study was measured in minutes. While this is not an obvious problem in our histogram comparisons (in comparing CDF and P-P plots) as well as computing test statistics, the discrete measure of our data will significantly impair the results. The second problem involves time spent in queues. The observations made for task duration are based solely upon the time a task arrived and when subsequently completed. No information is available on the amount of time a particular arrival may have spent in a

queue awaiting processing. With these caveats, we plot the histograms of the PCS and UFF task duration data with the parameter free exponential distribution at Figure 9 and Figure 10, respectively. Cell width in these histograms of the test data is fixed at 1.1 to insure the discrete data is accurately represented.



**Figure 9. PCS Duration with Exponential Distribution**

For the PCS duration data, the exponential distribution appears to produce a satisfactory fit, confirming our estimate of the behavior of task duration. Conducting a  $\chi^2$  goodness-of-fit test on this comparison yields a disappointing p-value of only .053, highlighting the effects of data idiosyncrasies on eventual hypothesis testing. For example, the absence of any seven minute task durations is a significant contributor to the high  $\chi^2$  statistic.



**Figure 10. UFF Duration with Exponential Distribution**

The UFF comparison also appears to support the selection of the exponential as our underlying theoretical distribution. A  $\chi^2$  test gives us an acceptable p-value of .253.<sup>4</sup>

Despite the suspect results obtained with the PCS task data, we can continue our examination of duration models with some confidence the underlying theoretical distribution is in fact exponential.

### C. INPUT DISTRIBUTION PARAMETER VALUES

The next step in developing our input distributions is to identify a methodology to select input parameters. From our examination of OPTEMPO, we will need three input parameters for each task representing arrival rates at HIGH/MID/LOW OPTEMPO. We must also determine the processing rates for task durations. The obvious suggestion is to use the database values for MIN/MODE/MAX interarrival times to determine scale parameters in dynamic and periodic tasks' arrival processes. This idea is not without justification, since the database estimates for interarrival times were initially obtained as expected MIN/MODE/MAX frequencies of arrivals. Conversion to MIN/MODE/MAX estimates for interarrival times is not strictly accurate. For example, if the estimate for expected maximum

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<sup>4</sup> We cannot reject the hypothesis that UFF duration data is from an exponential distribution at any reasonable critical value.

frequency for a particular task in a one hour period is six, it does not necessarily follow that the minimum interval for arrivals is ten minutes. To obtain the subject matter expert estimate for maximum arrivals we expect an average interarrival time of ten minutes. The database values are therefore valid estimates for calculating<sup>5</sup> the scale of the appropriate dynamic or periodic arrival distribution.

Formally stating our hypotheses concerning the use of the database in conjunction with our previous analysis of underlying distributions and the nature of C2 systems yields the following.

#### Dynamic Task Arrivals

$H_0$ : Let  $f(t)$  represent the distribution of interarrival times for any given dynamic task at time  $t$ , where  $\theta$ , represents the point estimate of the mean of an exponential distribution at time  $t$ . Then we can approximate  $f(t)$  as a composite function such that:

$$f(t) \sim \begin{cases} \text{EXP}(\theta_{\text{MIN}}) & , \quad t \in \text{High OPTEMPO} \\ \text{EXP}(\theta_{\text{MODE}}) & , \quad t \in \text{Medium OPTEMPO} \\ \text{EXP}(\theta_{\text{MAX}}) & , \quad t \in \text{Low OPTEMPO} \end{cases}$$

---

<sup>5</sup> For exponential distributions, our scale estimate is simply the database value. For Gamma distributions, the mean is given by  $\alpha\beta$  so with the database estimate of the mean (denoted by  $\theta_{\text{MIN/MODE/MAX}}$ ), we calculate  $\beta_{\text{HI/MID/LOW}} = \theta_{\text{MIN/MODE/MAX}}/\alpha$ .

### Periodic Task Arrivals

$H_0$ : Let  $g(t)$  represent the distribution of interarrival times for any given periodic task at time  $t$ , where  $\theta_t$  represents the point estimate of the mean of a Gamma distribution at time  $t$ . Then we can approximate  $g(t)$  as a composite function such that:

$$g(t) \sim \begin{cases} \text{GAMMA}(1, \theta_{\text{MIN}}) & , t \in \text{High OPTEMPO} \\ \text{GAMMA}(1.5, \theta_{\text{MODE}}/1.5) & , t \in \text{Medium OPTEMPO} \\ \text{GAMMA}(2, \theta_{\text{MAX}}/2) & , t \in \text{Low OPTEMPO} \end{cases}$$

### Task Duration

$H_0$ : Let  $h(t)$  represent the distribution of completion times for any given task (to include interactive tasks) at time  $t$ , where  $\theta_t$  represents the estimate of the mean<sup>6</sup> of an exponential distribution at time  $t$ . Then we can approximate  $h(t)$  such that:

$$h(t) \sim \begin{cases} \text{EXP}(\theta_{\text{MIN}}) & , \forall t \end{cases}$$

These three distributions ( $f$ ,  $g$ , and  $h$ ) incorporate our knowledge of C2 task behavior and underlying distributions. Successful testing of these distributions against our test

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<sup>6</sup> From our analysis of the nature of task durations, we should need only one point estimate for the mean. Selection of the Minimum expected completion time from the database is founded on the results obtained in our earlier examination of UFF and PCS duration data. Future data collection efforts should be conducted in this area to insure the conclusions reached in this paper (e.g., MIN value from database as estimate for task duration mean) are accurate, given the previously stipulated problems with task duration test data.

data will validate in principle our effort to refine the input process of C2NET, with the benefit of linking the force evaluation model's scenario into the activities of C2NET.

#### D. HYPOTHESES TESTING AND INPUT VALIDATION

We are now prepared to test our input distributions and associated parameters from the C2NET database against our test data. Table 1 summarizes the data values appropriate to each test. These values include the database estimate of the mean ( $\theta$ ), specified shape and scale parameters, and for comparison purposes, 95% confidence intervals for the scale parameter identified during our examination of the parameter free theoretical distributions.

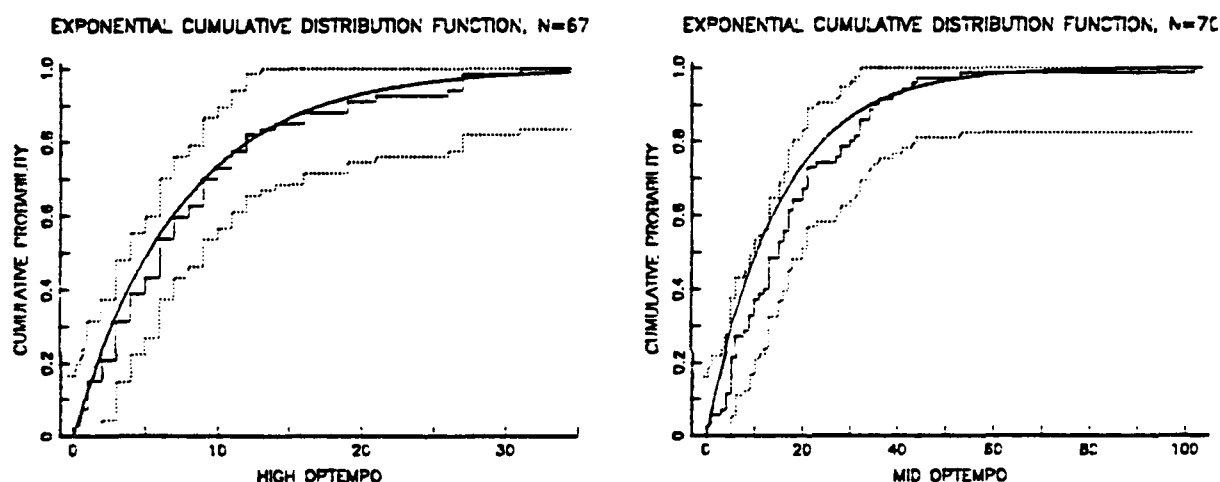
TABLE 1. DATA VALUES FOR HYPOTHESES TESTING<sup>7</sup>

	MEAN ESTIMATE	SHAPE PARAMETER	SCALE PARAMETER	95% C.I.
ARRIVAL PROCESS				
PCS MEDIUM	15.0	NA	15	14.6,23.3
PCS HIGH	7.5	NA	7.5	6.5,10.5
PIS LOW	60.0	NA	60	38.9,76.5
PIS MEDIUM	15.0	NA	30*	22.5,45.3
PIS HIGH	7.5	NA	7.5	7.02,17.9
UFF LOW	60.0	2	15*	6.3,20.6
UFF MEDIUM	15.0	1.5	10	3.6,14.3
UFF HIGH	7.5	1	15*	8.1,17.8
SERVICE PROCESS				
PCS (ALL OPTEMPO)	3.0	NA	3	1.62,2.39
UFF (ALL OPTEMPO)	3.0	NA	3	2.21,3.45

<sup>7</sup> Our hypothesis fails using the mean estimates from the database for tasks marked with "\*" in the Scale column. These failures are attributable to incorrect SME estimates, and new estimates that are not in conflict with empirical data are used as shown.

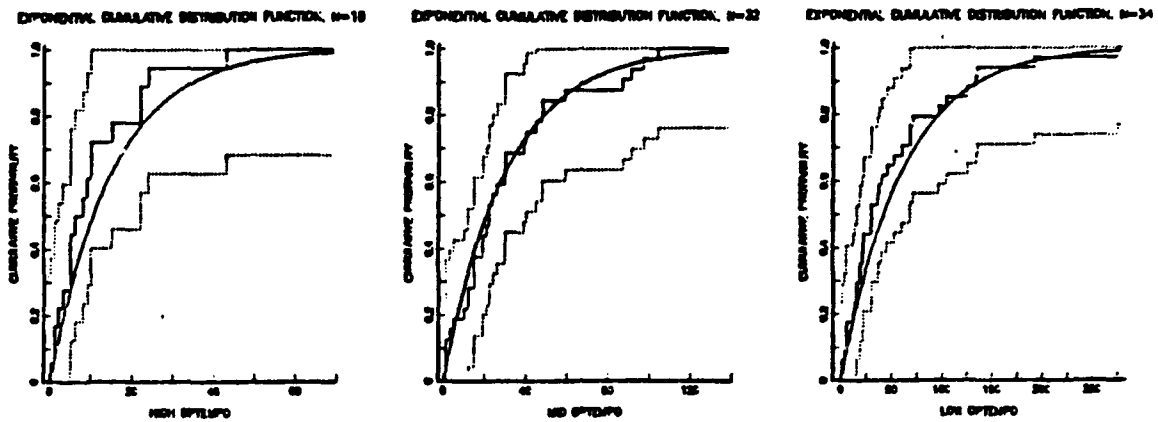


Again using the GRAFSTAT analytical tool and the data from Table 1, we compare the test data with the appropriate theoretical distribution. Figures 11-15 show the CDF plots from each of these comparisons. Results of  $\chi^2$  goodness-of-fit tests are in Table 2 for arrival processes and Table 3 for duration processes. Additional plots are at Appendix B (see Figures 18-22 for arrival processes and Figures 27-28 for task durations).



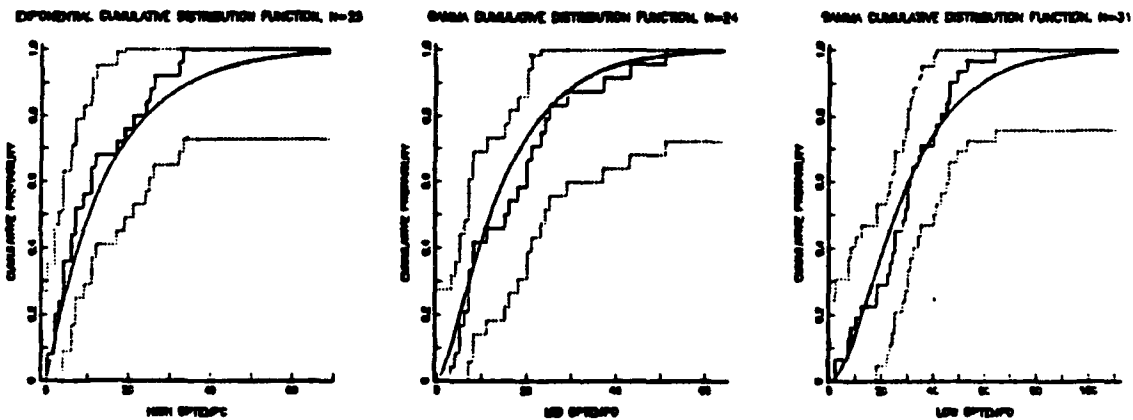
**Figure 11. PCS and Exponential Distribution CDF Plots**

At high OPTEMPO, the CDF plot shows a remarkably good fit. The medium OPTEMPO plot indicates that our estimate of the mean ( $\theta=15$ ) is above the true mean of the sample population. The generated p-value of 0.024 confirms our observation. The probable reason for this disparity is the presence of low OPTEMPO data in our test sample, lengthening the tail of the test data.



**Figure 12. PIS and Exponential Distribution CDF Plots**

All of these comparisons appear well behaved. In high and low OPTEMPO, there is some disparity, but in general the fits support the hypotheses regarding the  $f(t)$  distribution.



**Figure 13. UFF and Gamma Distribution CDF Plot**

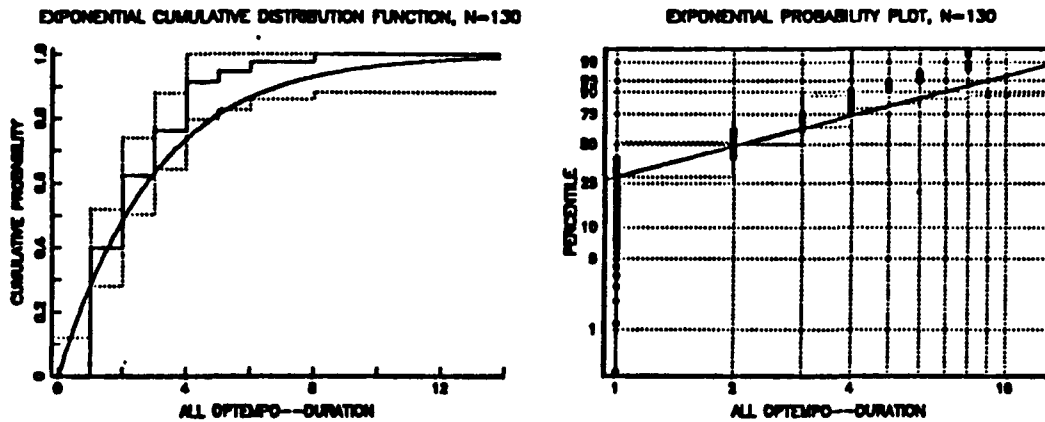
Using the corrected values for estimates of the mean result in the solid fits as shown in Figure 13. The right tail at high and low OPTEMPO are longer under our hypothesis of  $g(t)$  than in the test data, but not critically so. Note that the local modes so apparent in the histogram plots are transparent in the CDF plots at medium and low OPTEMPO. Test statistics generated from our examination of arrival sample data are as follows:

**TABLE 2. ARRIVAL HYPOTHESES TEST RESULT SUMMARY**

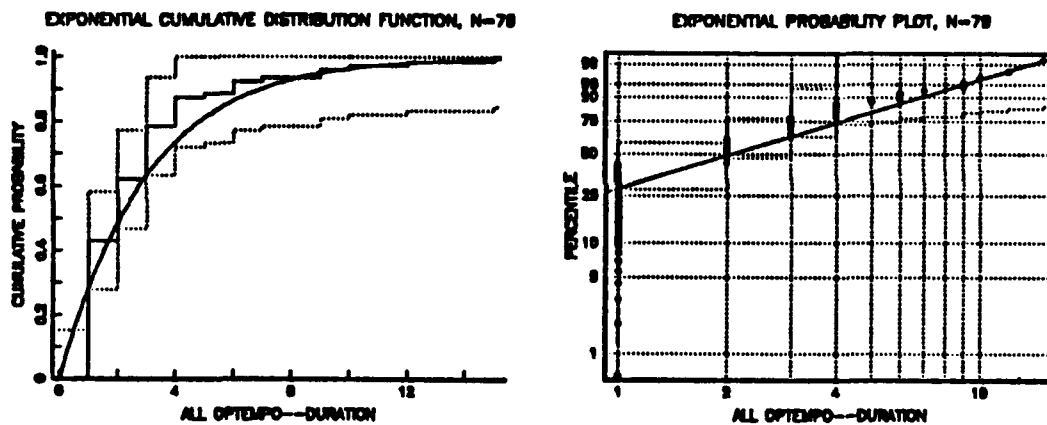
	TEST STATISTIC	df	p-VALUE
PCS MEDIUM	9.40	3	0.024
PCS HIGH	2.77	5	0.735
PIS LOW	2.41	3	0.491
PIS MEDIUM	1.01	3	0.797
PIS HIGH	1.25	2	0.536
UFF LOW	7.18	5	0.208
UFF MEDIUM	2.67	3	0.445
UFF HIGH	2.04	4	0.728

Similarly, we conduct comparative plotting and generate test statistics for the hypothesis regarding task duration. Figures 14 and 15 depict CDF and P-P plots for PCS and UFF duration, respectively.

The CDF and P-P plots of the PCS duration data with the exponential distributions highlight the problem of discrete data collection for the continuous service time process. The  $\chi^2$ -statistic generated from this comparison reflects the poor quality of the data with a p-value of 0.00022. The poor fit in the tails is the primary source of disparity.



**Figure 14. PCS Duration and Exponential Distribution Plots**



**Figure 15. UFF Duration and Exponential Distribution Plots**

Although we observe the same problem with the discrete test data as for the PCS plots, the UFF duration data appear to be well fit by the  $h(t)$  distribution and supports our null hypothesis.

**TABLE 3. DURATION HYPOTHESIS TEST RESULT SUMMARY**

	TEST STATISTIC	df	p-VALUE
PCS (ALL OPTEMPO)	28.03	6	0.00022
UFF (ALL OPTEMPO)	7.86	5	0.16400

The *p*-values generated by tests, except where noted, support the hypothetical distributions *f*, *g*, and *h*. Disparities between these distributions and the test data are primarily attributable to errors in the data collection methodology (affecting primarily the duration tests), errors in the database estimates (UFF at low and high OPTEMPO and PIS at medium OPTEMPO), and idiosyncracies in the test data. With the correct estimates of mean and accounting for the other sources of error, the test results strongly support the validity of our procedure for modeling C2 task input processes in C2NET.

Our analyses have thus far centered on improving the C2NET model through enhancing the input process. Our next concern must be to examine the C2NET output and how it is used in the force evaluation model. In the next chapter we will conduct this examination, again with the intent of identifying disparities and analyzing potential solutions to reduce the impact on model performance.

### III. C2NET OUTPUT VALIDATION

In actual usage, of course, models are typically applied to situations not strictly represented. This may be done deliberately, and one then speaks of ingeniously "gimmicking" the model. If, for example, a land warfare model does not strictly represent supporting air power, but does represent artillery, then in application air power may be represented as artillery. Strictly speaking, however, any validation of such a model in such an extended application should include validation of the "rules" of extension. How, that is, does one represent air power as artillery? How are the numerical values of the inputs to be selected? (Thomas, 1989, pg. 265)

Validation of C2NET prompts us to ask; how does one represent command and control in a force evaluation model? Specifically, how do we use the output from C2NET as input into VIC to simulate C2 effects on Division/Corps operations? This is a significant challenge given the deterministic design of VIC and the imprecise nature of C2. The first step in the validation of C2NET output is analysis of the "rules of extension" currently in use to approximate C2 in VIC. We will then examine a new method for portraying C2 effects through application of fuzzy set theory. Finally, we will construct and discuss new rules of extension using our fuzzy methodology to apply C2NET output within VIC.

## **A. ANALYSIS OF CURRENT C2NET OUTPUT PARAMETERS**

C2NET is designed to measure the responsiveness of C2 systems with VIC used to determine the impact of this responsiveness on force effectiveness. Currently, C2 effects input to VIC consist entirely of delay times influencing execution of strike missions<sup>1</sup> and maneuver orders as output from C2NET. These delay inputs are summarized as follows:

C2NET outputs (queuing and processing delays) are inputs to VIC. Three types of delays will be adjusted in VIC-- fusion, decision, and execution. Fusion delays may be entered for each sensor -- without regard to the echelon supported. Decision delays are of two types -- artillery and maneuver. VIC will accept artillery decision delays at each echelon -- corps, division, and brigade. A single maneuver decision delay may be entered at each dwell point along a maneuver unit's path -- corps, division, and brigade. Execution delays may be entered for artillery by attack means. (Ramaden, 1991, pg. 7-1)

The delays represent the following:

- Fusion Center delays in processing sensor reports into strike target nominations.
- Decision delays in the staff process for processing target nominations and movement orders.
- Execution delays from receipt of strike mission order to rounds on the way.

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<sup>1</sup> "Strike" missions include tube and missile artillery fires, attack helicopters (AH), and/or USAF close air support/battlefield interdiction (CAS/BI) sorties.

The fusion and execution delay factor inputs into VIC are estimates from the appropriate service schools. Decision delays are calculated from appropriate process delays observed in C2NET runs. When combined, these delay factors determine the maneuver and request for strike mission asset delay factors to be entered into VIC.

Two problems are associated with the current method by which C2 responsiveness is approximated in VIC. The first, most obvious problem is the aggregation of delays into single point estimates by echelon for maneuver forces and delivery means for strike assets. As discussed previously in our analysis of input processes, we expect the C2 system workload to vary with tactical intensity producing variations in system responsiveness. The second problem was summarized by Doctor La Rocque, Director of the TRADOC Operations Analysis Center at Fort Leavenworth, "Command and Control is not just delay times." While the aggregation of times required to complete key C2 tasks is a valid approximation of decision delay, delays alone are insufficient approximations of C2 effects in a force evaluation model. In particular, VIC as a deterministic model with scripted maneuver fails to simulate the full effect maneuver and strike mission delays have when combined with all other mission attributes.



To demonstrate, we will examine the concept of artillery effectiveness. Any given artillery mission's effectiveness is dependent on a number of variables as represented in Figure 16. The decision process, isolated from the other explanatory variables, fails to accurately simulate the compound effects. For example, a ten minute delay between call for fire and rounds on the way may be satisfactory for a slow moving target in open terrain where observation may be maintained and fires accurately adjusted. However, if the target is moving quickly or in broken terrain, this delay may cause the fires to be wholly ineffective. VIC in its current form does not account for these additional factors given a delay value; it simply delays the application of attrition for the required time. Similar problems are encountered with maneuver delays scripted into "dwell points" for moving forces.

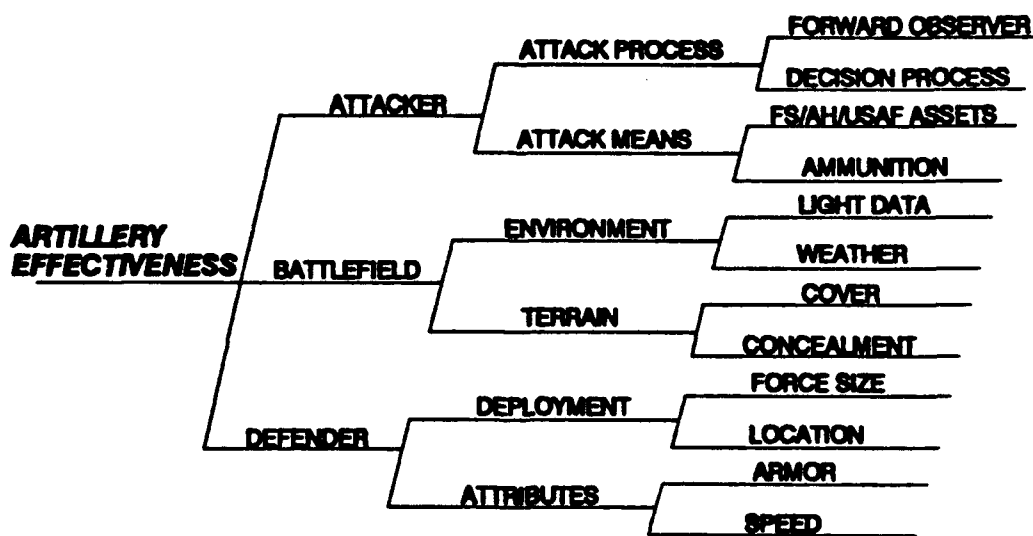


Figure 16. Conceptual Hierarchy--Artillery Effectiveness

## **B. EXAMINATION OF SOLUTIONS TO OUTPUT DISPARITIES**

Our previous analysis of the C2NET input process will serve as the basis for solving the aggregation problem. The current input process relies on single input distributions for each task by echelon. Thus the resolution of the output is reasonable given the aggregation of the input. By increasing the level of detail in C2NET to each unit over time (represented by OPTEMPO timelines for each unit over the life of the scenario) as proposed, we gain the additional benefit of output for each unit over time. This segregation will improve our approximation of the effects of C2 responsiveness within VIC significantly, as decision delay factors are now linked to the tactical scenario for each unit and not simple point estimates by echelon or attack means for the entire scenario.

Accurately capturing the effects of C2 responsiveness in our force evaluation model, beyond delay times, is resistant to a simple solution. As in our artillery effectiveness example, delays in the decision and execution of a fire mission will certainly have an effect on its outcome. Our imprecise knowledge of the cumulative effect of responsiveness with other explanatory variables suggests a potential methodology to define C2 effects in a force evaluation model--fuzzy set theory.

## 1. Basics of Fuzzy Set Theory

In classical normative decision theory the components of the basic model of decision making under certainty are taken to be crisp sets or functions. By crisp we mean dichotomous--that is, of the yes-or-no type rather than of the more-or-less type. The set of actions is as precisely defined as the set of possible states (or the state) and the utility function is also assumed to be precise.

Fuzzy set theory provides a strict mathematical framework...in which vague conceptual phenomena can be precisely and rigorously studied. It can also be considered as a modelling language well suited for situations in which fuzzy relations, criteria, and phenomena exist. (Zimmermann, 1987, pp. 10-11)

Building upon this difference between classical and fuzzy sets, Zimmermann formally defines a fuzzy set as follows:

If  $X$  is a collection of objects denoted generically by  $x$  then a fuzzy set  $\tilde{A}$  in  $X$  is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$$

$\mu_{\tilde{A}}$  is the membership function that maps  $X$  to the membership space  $M$  and  $\mu_{\tilde{A}}(x)$  is the grade of membership (also degree of compatibility or degree of truth) of  $x$  in  $\tilde{A}$ .

Fuzzy set theory appears ideally suited to assist us in defining and quantifying the imprecise nature of C2 effects in an evaluation model. Applying this definition to our artillery effectiveness problem will give us a clearer picture of the utility of fuzzy set theory in the C2 responsiveness problem. For example, we want to classify the performance of artillery in direct support of maneuver units. One indicator

of the effectiveness of artillery is the delay time between request for fire and rounds on the way. Let  $X=\{1,2,3,\dots,10\}$  be the set of possible delay times (in minutes). Then the fuzzy set "effective artillery fire against a stationary target" may be described as

$$\tilde{A}=\{(1,1), (2,.98), (3,.95), (4,.9), (5,.8), (6,.65), (7,.5), (8,.25), (9,.1), (10,0.0)\}$$

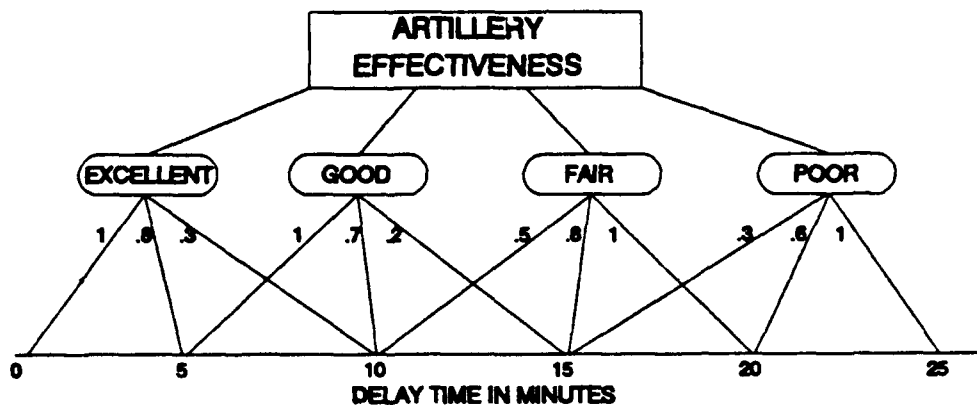
In this example, a one minute delay is always mapped into the fuzzy set "effective artillery fire," while a seven minute delay has a 0.5 degree of membership to this set. Note that the membership function in this example is monotone. This is not a requirement and most such functions are not monotone (it is therefore inappropriate to consider this function as some type of cumulative distribution function or attempt to construct a corresponding probability function). To completely describe "effective artillery fire given target speed" in this example we would also be interested in constructing the fuzzy sets for slow, moderate, and fast moving ground targets. This allows us to generalize artillery effectiveness for all target types; whether an enemy defense or threat tank company halted in a minefield, an infantry platoon advancing under small arms fire (slow), a motorized rifle battalion attacking cross-country (moderate), or a truck convoy on a highway (fast).

## 2. Application of Fuzzy Set Theory in C2

Before proposing a solution to the C2 responsiveness issue utilizing fuzzy set theory, we need to refine our model. The fuzzy sets as presented in our model consider only one additional explanatory variable (target speed). We must also keep in mind that the delay times provided from C2NET are only *approximations* of average system performance at any given time in the battle. Given the deterministic nature of VIC, this is quite satisfactory (our alternative is to randomly sample some sort of empirical distribution produced by C2NET for delay times--unacceptable in the VIC environment).

To account for the contributions of other (albeit less significant) variables and the approximations made in delay factors, we will introduce the additional concept of "fuzzy restrictions." In our artillery example, these restrictions are identified as the following: Ineffective, Poor, Fair, Good, and Excellent Effectiveness. These restrictions will serve to aggregate the membership functions of the fuzzy sets to a reasonable level of resolution given our imprecise knowledge about the system. We can also establish penalties to the effectiveness (attrition coefficients) of artillery missions associated with each value of effectiveness, with eventual application to portraying C2 effects in the evaluation model. In this extension of the artillery problem, any given delay time may be a member of one or more

restriction variables (e.g., a certain delay time that has a 0.5 degree of membership to the "Excellent" category is at the same time almost certainly in the "Good" category). The next step is to determine for each delay time ( $x$ ) the corresponding degree of membership ( $\mu_A(x)$ ) into each fuzzy restriction (where membership values will differ from the fuzzy set examined previously). At Figure 17, there is an example of how delay time memberships are mapped into the fuzzy restrictions Excellent through Poor.



**Figure 17. Artillery Effects with Fuzzy Restrictions**

Analyzing the fuzzy restriction "Good Artillery Effectiveness," for example, indicates that a delay of five minutes will always be a member of this set. At ten minutes there is a 0.7 degree of membership that this delay will result in good effectiveness, while at 15 minutes the corresponding mapping to a Good result is down to 0.2.

The final step in developing a satisfactory method for modeling C2 responsiveness in a force evaluation model is to design a *nonstochastic* function mapping key variables into the fuzzy restrictions. The recommended method requires we depart from the fuzzy construct for mapping the effectiveness function in VIC. Doctor Pat Allan of the Rand Corporation suggested this method in a phone conversation with the author (19 August, 1992), as an adaptation of the Rand "Able Tables." This method is based on our existing model consisting of a fuzzy variable ("Artillery Effectiveness") with restrictions ("Good," "Poor," ...) and corresponding mappings of the degree of membership for each base variable into these restrictions (e.g., a five minute delay has a degree of membership of 0.8 in the set of "Excellent Artillery Effectiveness" and 1.0 to the set "Good Artillery Effectiveness"). Using this model, the analyst must then set a *minimum degree of membership constraint* (or  $\alpha$ -level). The  $\alpha$ -level limits the possible restrictions for mapping; the degree of membership must equal or exceed this level to be mapped into a restriction. Given the set of possible mappings after application of the  $\alpha$ -level, the delay times are subsequently mapped into the best possible restriction. In the artillery example, if we set our  $\alpha$ -level at 0.5, then a five minute delay is mapped into "Excellent," and a ten minute delay into "Good" (since the 0.3 degree of membership into "Excellent" is less than our  $\alpha$ -level and the "Good" variable is preferable to "Fair"). Thus for each delay

time and associated explanatory variable(s) in VIC, there is only one corresponding effectiveness value satisfying our requirement for constructing a nonstochastic function mapping key variables into fuzzy restrictions.

Continuing with the artillery example, in Table 4 we identify the value of artillery mission effectiveness given the explanatory variable "target speed" (known to VIC for each mission) and the current total delay time from request to fire (from C2NET output). The responses in the table range from Excellent (E) to Ineffective (I). The numbers in parentheses indicate the portion of the original attrition coefficient applied under a given response--e.g., a Good mission will apply 90% of its computed attrition coefficient. This type of response matrix is easily input into VIC and will not require any significant restructuring of existing VIC decision tables. The largest investment involved in implementing this methodology is in performing the sensitivity analysis required to set penalty values at the appropriate levels.



**TABLE 4. ARTILLERY EFFECTIVENESS MATRIX**

DELAY (MIN)	STATIONARY (0 km/hr)	SLOW (< 10 km/hr)	MODERATE (< 25 km/hr)	FAST (> 25 km/hr)
1	E (1.0)	E (1.0)	E (1.0)	G (0.9)
2	E (1.0)	E (1.0)	G (0.9)	F (0.5)
3	E (1.0)	G (0.9)	G (0.9)	F (0.5)
4	E (1.0)	G (0.9)	G (0.9)	P (0.3)
5	E (1.0)	G (0.9)	F (0.5)	P (0.3)
6	G (0.9)	F (0.5)	F (0.5)	I (0.0)
7	G (0.9)	F (0.5)	P (0.3)	I (0.0)
8	G (0.9)	F (0.5)	P (0.3)	I (0.0)
9	G (0.9)	P (0.3)	P (0.3)	I (0.0)
10	F (0.5)	P (0.3)	I (0.0)	I (0.0)

**KEY:**

E - EXCELLENT

G - GOOD

F - FAIR

P - POOR

I - INEFFECTIVE

**NOTE:** The values in parentheses reflect adjustments to a fire mission's attrition coefficient when it has the given effectiveness. (e.g., INEFFECTIVE missions have no effect)

At first glance, this methodology appears to entail a significant effort to produce a simple response table. However, we must keep in mind that this simple table is an approximation of the corresponding fuzzy set. Only through analysis of the response as a fuzzy variable are we able to define, quantify, and eventually approximate its behavior accurately. The additional benefit of this methodology is its generality--the potential for solving other ill-defined problems.

Suppose we are interested in comparing the "Agility" of friendly forces equipped with different C2 hardware in an evaluation model using C2NET output. The procedure for this

comparison using fuzzy set theory would follow that used in our artillery example:

- Determine contributing key C2 tasks
- Calculate delays using C2NET
- Identify significant explanatory variable(s) from the evaluation model
- Construct fuzzy set(s) with restrictions and corresponding penalties<sup>9</sup>
- Approximate the "Effectiveness Matrix" using fuzzy sets, restrictions, and resulting penalties over the range of possible delays and other explanatory variables
- Input the Matrix and run evaluation model (may require sensitivity analysis before test run)

Our new model of delays and corresponding penalties will significantly reduce the disparity between the real system and how we approximate the effects of C2 in the force evaluation model. How much better is the chance of a successful air strike or artillery mission against an attacking enemy slowed or stopped by an obstacle? Likewise the model now captures the difficulty of putting effective fires on a target moving unhindered on a road. Historically, most fire missions shot against such a target will fall on empty ground. Proper tactics are rewarded; construction of engagement areas with obstacles (natural and emplaced) will

---

<sup>9</sup> Penalties can be designed to effect attrition coefficients, movement routes/rates, resupply, .... The analyst must determine what is appropriate and workable within the evaluation model to reflect performance.

slow and constrict enemy maneuver into friendly direct and indirect fires and in deep attacks, interdiction of bridges and road junctions will stop or slow reinforcing columns to enhance artillery, missile, and air strike conditioning of these forces.

Similar applications of fuzzy set theory to maneuver and fusion delay are possible using the procedure as outlined previously. Since maneuver is scripted in VIC, penalties associated with maneuver effectiveness will probably entail extension of the original scenario to include additional paths from each dwell point, allowing units to select the path taken based on efficiency--e.g., at poor effectiveness, the unit moves to indecisive terrain.

#### C. THE FINAL MODEL

The proposed changes and enhancements to C2FAM are all designed to increase model fidelity and decrease disparity between the real system and how we approximate this system. By introducing the concept of operational tempo for each unit by battlefield operating system, we can quantify the tactical scenario of the evaluation model into a form usable by C2NET. Identifying the correct underlying theoretical distribution for each key C2 task and using the existing subject matter expert estimates to define shape and scale parameters under the different levels of operational tempo will allow us to more accurately simulate the task arrival and duration

processes. We gain the additional benefit of higher resolution output by segregating C2 responsiveness by unit over the life of the scenario. Finally, by examining the output as a fuzzy phenomena, we will be able to make nonstochastic statements about system effectiveness (as a result of the C2 process) within the evaluation model. These statements with corresponding adjustments to the behavior of forces will enhance the quality of the model's performance by better capturing the effects of C2 responsiveness on operational capabilities.

The net result of this proposal, if implemented, will be a model truer to the real system through better approximation, higher resolution, and an efficient model interface. Implementing some portions of this proposal will be simple (e.g., changing the distributions for arrival and task duration in C2NET); most of it will not be simple. The next chapter discusses recommendations and potential problems in modifying C2FAM (that is, C2NET, the force evaluation model, and linkage mechanisms) in accordance with this proposal.

#### **IV. AREAS FOR FUTURE STUDY**

##### **A. INPUT PROCESS VALIDATION**

From our analysis of C2NET we know there is much work to be done to improve the input process. This section identifies areas of potential trouble to Army analysts in implementing this proposal and recommends procedures to solve these problems.

The key to linking the tactical scenario of the evaluation model into C2NET is the concept of operational tempo. Creating these OPTEMPO timelines for each scenario of interest will challenge the analyst due to the number required (one for each unit by operating system) and lack of clear definitions for "High," "Medium," and "Low" OPTEMPO for each battlefield operating system (BOS). A simple heuristic approach to constructing these timelines is based on the concepts of Air-Land Operations (ALO). ALO identifies four stages of tactical/operational combat:

- Stage 1: Detection and Preparation Phase
- Stage 2: Establish Conditions for Decisive Operations
- Stage 3: Decisive Operations
- Stage 4: Reconstitution

We state without proof that OPTEMPO for each BOS is dependent on the ALO stage a unit is in (e.g., a unit conducting decisive operations would have a higher maneuver tempo than a unit undergoing reconstitution). We can therefore approximate all BOS OPTEMPO timelines if we know when a unit is in which ALO stage and the corresponding OPTEMPO values for each stage. This approximation will simplify the timeline construction process, as the scenario developers know which stage each unit is in for a scripted model (like VIC). Table 5 indicates how BOS tempo may be approximated by ALO stage.

**TABLE 5. BOS OPTEMPO BY ALO STAGE<sup>10</sup>**

<b>S T A G E</b>	<b>MVR</b>	<b>INTEL</b>	<b>FIRE SPT</b>	<b>CSS</b>	<b>AIR DEFENSE</b>
<b>1</b>	<b>L</b>	<b>H</b>	<b>M</b>	<b>L</b>	<b>L</b>
<b>2</b>	<b>M</b>	<b>M</b>	<b>M</b>	<b>L</b>	<b>M</b>
<b>3</b>	<b>H</b>	<b>H</b>	<b>H</b>	<b>M</b>	<b>H</b>
<b>4</b>	<b>M</b>	<b>L</b>	<b>L</b>	<b>H</b>	<b>M</b>

---

<sup>10</sup> Combat Service Support (CSS) and Air Defense are new modules under development for incorporation into C2NET during FY93.

The other major obstacle to our input process improvement effort is database validation. As demonstrated in our hypothesis testing, the subject matter expert estimates for the arrival process are not necessarily accurate and there remains some question as to the proper parameter to use in the service process. However, now that we are confident we have identified the correct underlying theoretical distributions, it is possible to test the SME estimates against observed data of the real system.

Just as model validation is an ongoing process, so should database validation be a continuous effort. Army analysts involved with C2NET should continue to gather test data from other analytical agencies (e.g., the ARI studies and Force Level Control System experiments conducted by the MITRE Corporation), screen these data for applicability to C2NET tasks, and then conduct hypothesis testing on the database parameters. Prior to testing, database parameters can be used in their current form, with the caveat that C2NET results are only valid in use with comparative studies, where database errors will tend to have equal effects.

#### **B. OUTPUT PROCESS VALIDATION**

In developing C2 effectiveness inputs to the evaluation model the analyst must resist the temptation to go directly from identification of key explanatory variables to construction of a response table. The approach discussed in

Chapter III assures the analyst that the effects of key variables are correctly defined and the results input into the evaluation model are accurate approximations of C2 effectiveness.

The most significant challenge to the analyst in this process is correctly constructing the fuzzy set(s) with corresponding restrictions and penalties. Subject matter experts should be involved in this process, using their empirical knowledge of the system to map the degrees of membership for the base variables into the fuzzy restrictions. In the artillery example, it would be proper to ask artillery instructors at the Command and General Staff College, based on their personal observations, questions relating to mission effectiveness given processing delay times and target speed. Their answers to these questions could then be used to construct SME estimates of the fuzzy sets. Where possible, these estimates should then be tested against real system performance data (e.g., National Training Center observations, Desert Storm,...).

The final step in our approach, inputting the response table into the evaluation model and running the model, is also a challenge. As a scripted model, changing the attributes of the model (attrition coefficients, maneuver paths, resupply rates, ...) can have unexpected consequences. Additionally, enemy forces operating without penalties (from less than perfect C2) will have an unrealistic advantage.



To reduce the impact of the first problem, the analyst in conjunction with the scenario developer, will need to conduct an analysis of the model after C2 response tables are input. This will initially represent significant work, as the penalties may need adjustment to better approximate C2 effects in the model. The second problem will require either development of a Threat C2NET or some more generalized methodology to account for C2 effects on the operational capabilities of enemy forces.

### C. CONCLUSIONS

The intent of this proposal is to identify significant disparities between the Command and Control Functional Area Model and the real world performance and effects of command and control systems, and recommend potential solutions to reduce or eliminate these disparities. Through examination of the C2FAM input and output processes, this thesis has met its goal. However, even with successful implementation of all the recommendations as proposed, the validation process for C2FAM will not be completed. New command and control systems and changes in warfighting concepts will require a continuous effort by Army analysts to insure that the C2FAM (C2NET, evaluation model, and linkage mechanisms) remains a valid, worthwhile model into the future.

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## APPENDIX A. BASICS OF C2NET

The command and control network model (C2NET) is a low resolution model representing essential command and control (C2) tasks performed by Corps, Division, and Brigade command, control, communications, and intelligence (C3I) systems (Command Posts, Fire Direction Centers, Intel Fusion Centers, etc.). C2NET was developed as a direct result of the need for a command and control functional area model (C2FAM) to assist Army analysts in conducting trade-off analyses as fiscal constraints grow tighter, as well as explore new emerging warfighting concepts. Output (measures of performance) from the C2NET performance model is intended to be used as C2 input parameters for force effectiveness models. To date, C2NET output has been used with the Vector-in-Commander (VIC) effectiveness model to study C2 responsiveness and alternative Division command post configurations.

C2NET was built in FY91 by Potomac Systems Engineering (PSE) for the TRADOC Analysis Command - Operations Analysis Center (TRAC-OAC), Fort Leavenworth, KS. C2NET is based on the Modeler programming tool. Modeler is an interactive tool written in the "C" programming language that uses a graphical interface to produce Stochastically Timed Attributed Petri Net (STAPN) event-stepped simulation models. A Petri Net is a bipartite graph consisting of queues and servers connected by

arcs. Petri Nets also employ primitive symbols and objects known as "tokens" (customers) that accumulate in queues and are consumed when served.

For descriptive purposes, the Petri Net (C2NET) closely resembles an Open Queuing Network, complicated by:

- Before service can occur, all queues into the server must be occupied.
- Customer generation can occur internal to the network (each service can generate more than one customer, with each arc from server to queue generating a customer).

Despite these caveats, many of the basic modelling considerations of an open queuing network apply to C2NET, and must be addressed before C2NET is validated (such as interarrival rates [task generation], service rates, and corresponding theoretical distributions).

The Modeler programming tool allows the C2NET post-processor to produce output using four measurable quantities:

- Probability. The model probabilities for each flow of tokens across the arcs connecting each server (or queue).
- Rate. Transition frequencies from place to place during the simulation.
- Occupancy. The average number of tokens in each node during the simulation.
- Delay. The time between transitions.

The key tasks currently represented in C2NET include those from the maneuver (MVR), intelligence and electronic warfare

(IEW), and fire support (FS) battlefield operating systems. The combat service support and air defense operating systems are under study for planned incorporation into C2NET. A list of key tasks currently modeled (by echelon and BOS) is at Table 6. Additional information may be obtained regarding C2NET by writing the Director, Command and Control, Communications, and Intelligence Studies and Analysis Directorate (C3I SAD), ATTN: ATRC-FS, Fort Leavenworth, KS, 66027-5200.

**TABLE 6. C2NET KEY TASK LIST**

**ECHOLON MODELED:**

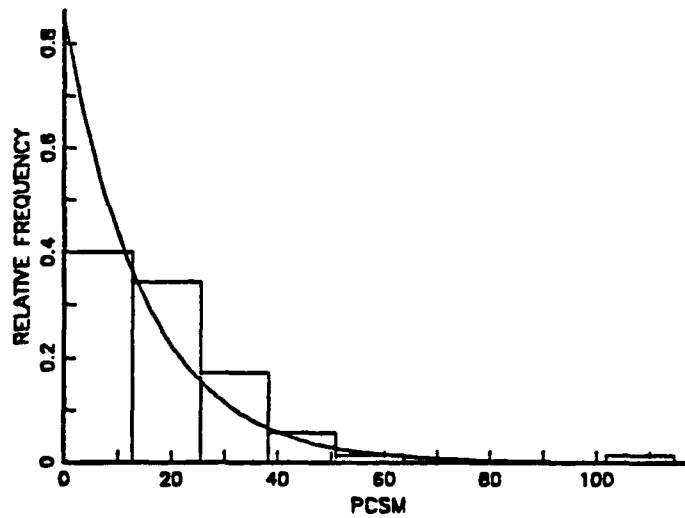
<b>KEY TASKS</b>	<b>C</b>	<b>D</b>	<b>B</b>
<b>MANEUVER TASKS</b>			
Assess Tactical Operations		X	X
Develop Attack Criteria	X	X	X
Develop Commander's Planning Guidance	X	X	X
Develop Concept of Deep Operations	X		
Develop Engineer Priorities	X	X	X
Develop Force Alternative Courses of Action	X		
Develop Subordinate Command Missions	X	X	X
Evaluate Unit Status	X	X	X
Prepare FRAGOs	X	X	X
Prepare Operational Orders	X	X	
Prepare Operations SITREP	X	X	
Prepare Warning Order	X	X	X
Process FRAGO		X	X
Select Force Course of Action	X	X	X
Update Status of Adjacent Forces	X		
Update Status of C3CM Situation	X		
Update Status of Mobility/Counter-Mob Situation	X		
Update Status of Engineer Situation	X		
Update Status of Friendly Forces	X	X	X
Update Status of OPSEC Situation	X	X	
Update Status of Plans and Orders	X		
Update Status of Tactical Situation	X	X	X

**TABLE 6. C2NET KEY TASK LIST (CONTINUED)**

<b>FIRE SUPPORT TASKS</b>			
Coordinate Army Aviation Support (CAS)	X	X	
Coordinate JAAT for Close Operations		X	
Coordinate Tactical Air Support	X	X	X
Develop Priority of Fires	X	X	X
Process Requests for Aviation Support	X	X	X
Process Requests for Close Air Support	X	X	X
Process Requests for Immediate Artillery Support	X	X	
Process Requests for Immediate MLRS Support	X	X	
Process Target Nomination (FS)	X	X	X
Process Target Nomination (Sensor)	X	X	X
Update Status of Air Mission Results	X	X	
Update Status of Air Support Assets	X	X	
Update Status of Artillery Units	X	X	X
Update Status of MLRS Units	X		
Update Status of Conventional Targets	X	X	X
Update Status of Conventional Fire Requests	X		
Update Status of Conventional Strike Results			X
Update Status of FS Order of Battle	X		
Update Status of FS Situation	X	X	X
Update Status of JAAT Operations	X	X	X
Update Status of Joint Air Support Operations	X	X	
<b>INTELLIGENCE &amp; ELECTRONIC WARFARE TASKS</b>			
Process Critical Deep Operations Events	X		
Process Critical Situation Reports	X	X	X
Process Intelligence Spot Report	X	X	X
Update Status of Current Intelligence	X	X	X
Update Status of Deep Operations Situation	X		
Update Status of Target Acquisition Capability	X	X	

# APPENDIX B. FIGURES TO SUPPORT HYPOTHESIS TESTING

EXPONENTIAL DENSITY FUNCTION, N=70



EXPONENTIAL PROBABILITY PLOT, N=70

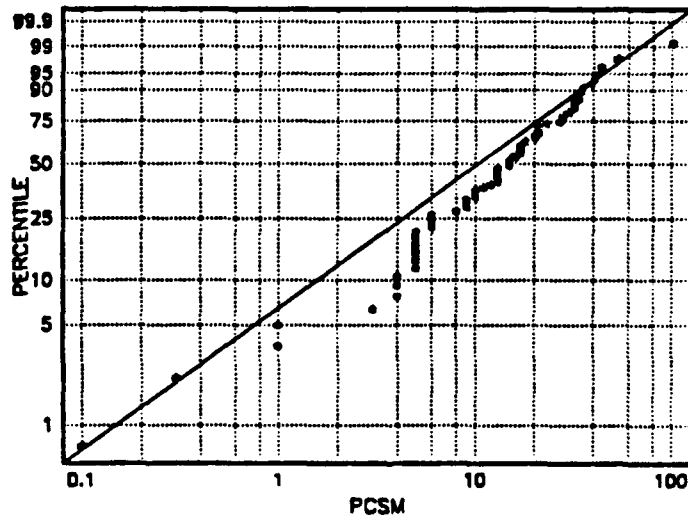


Figure 18. PCS Mid OPTEMPO [Exp( $\theta=15$ )]



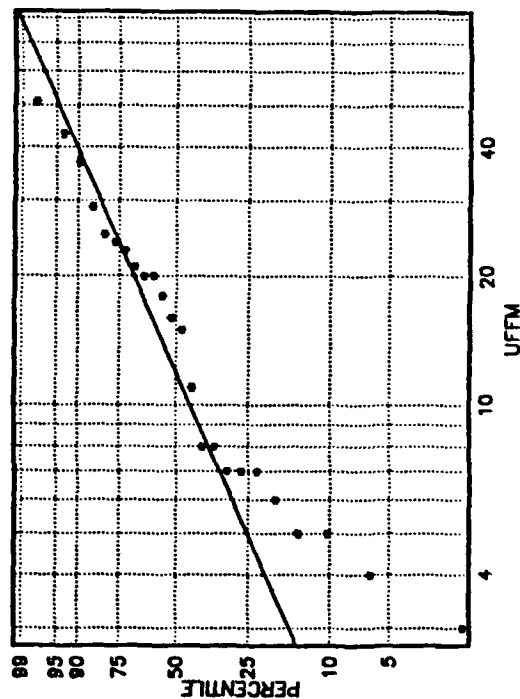
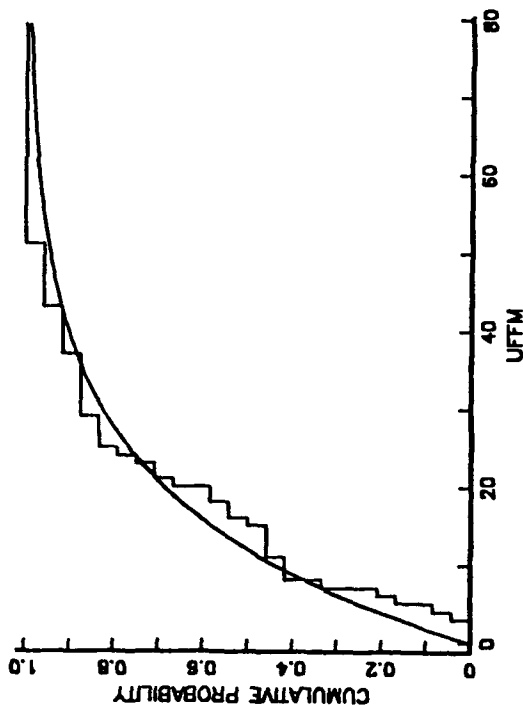
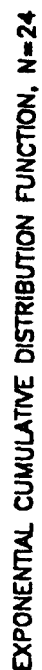
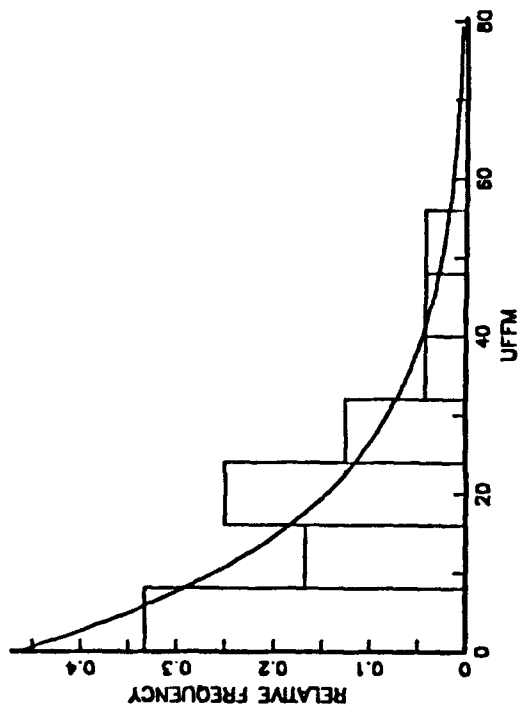
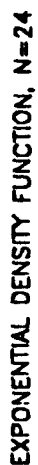








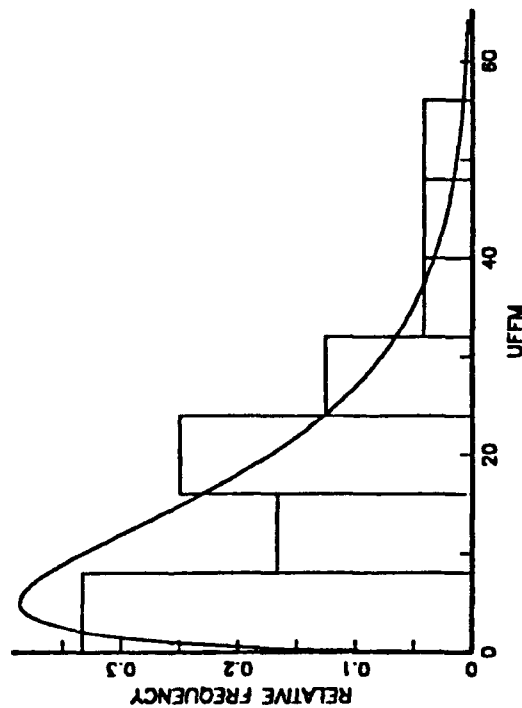




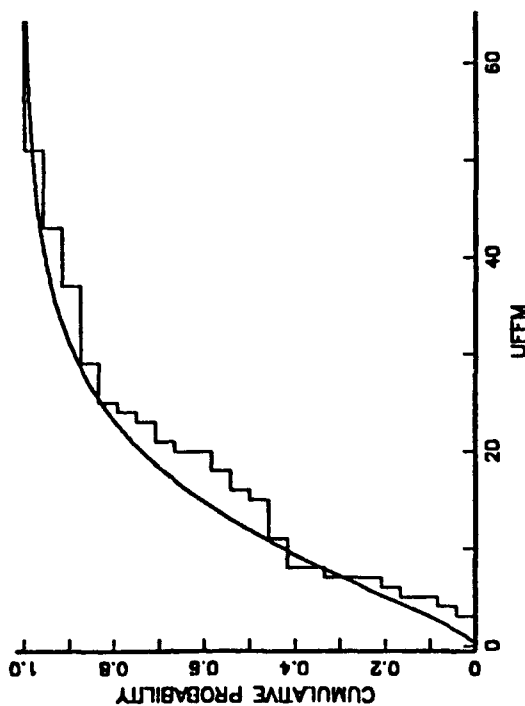
**Figure 24. UFF Mid OPTEMPO [Gamma( $\alpha=1$ ,  $\beta$ =FREE)]**



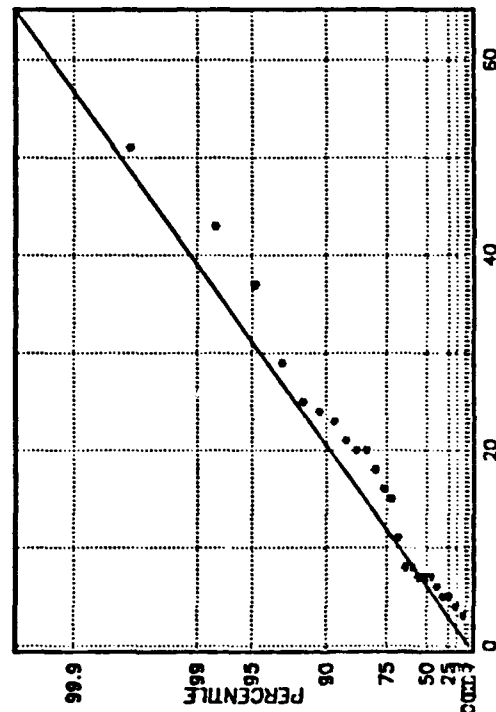
GAMMA DENSITY FUNCTION, N=24



GAMMA CUMULATIVE DISTRIBUTION FUNCTION, N=24



GAMMA PROBABILITY PLOT, N=24



ANALYSIS OF GAMMA DISTRIBUTION FIT

DATA : SUPP  
 SELECTION : ALL  
 X AXIS LABEL : UFFM  
 Y AXIS LABEL : REL FREQ  
 REGRESSION : NONE  
 PERCENTILES : 1  
 PARAM SPACE : ALPHA = 1.0 AND BETA = 10

PARAMETER ESTIMATE  
 ALPHA 1.0  
 BETA 10

CONFIDENCE INTERVALS OF  
 PARAMETER ESTIMATES  
 NOT AVAILABLE

LOW LIMITING POSITION AT X=0 = 99.788

MEAN : 12.000  
 STD DEV : 12.000  
 VARIANCE : 144.000  
 SKEWNESS : 3.000  
 KURTOSIS : 9.000

BASED ON DISCRETE OF 7 DIGIT INTERVALS

PERCENTILES : SAMPLE FITTED  
 25 10.0 0.0000  
 50 12.0 0.0000  
 75 14.0 0.0000  
 90 16.0 0.0000  
 95 18.0 0.0000  
 99 20.0 0.0000

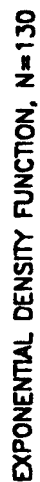
COEFFICIENTS OF FIT TESTS  
 CHI-SQUARE : 0.076  
 DEG FREEDOM : 23  
 P-VALUE : 0.999  
 WILCOX-SHAPIRO : 0.0000  
 SLOTTEN : 0.0000  
 SLOTTEN-W : 0.0000  
 SLOTTEN-W : 0.0000  
 SLOTTEN-W : 0.0000  
 SLOTTEN-W : 0.0000

CHI-SQUARE COEFFICIENTS OF FIT TABLE

LOWER TAIL	UPPER TAIL	CHI-SQ	DEG-F	P-VALUE
0.001	0.999	0.076	23	0.999
0.01	0.99	0.145	23	0.999
0.05	0.95	0.485	23	0.999
0.10	0.90	0.576	23	0.999
0.25	0.75	0.833	23	0.999
0.50	0.50	1.000	23	0.999

Figure 26. UFF Mid OPTEMPO [Gamma( $\alpha=1.5$ ,  $\beta=10$ )]





75

ANALYSIS OF REPORTS AT 0130Z 01/01/78

```
DATA      ;
SLENTRY=0; ALL          .c{pccc.cccc}
N ABIS LABEL; TARE SUMATION {BIBWZD}
SAMPLE SIZE; TDE
SUBROUTINE; SONE
PRECISION; 1
COMP METHOD; MAXIMUM LIKELIHOOD
ENDAT;
```

PARAMETER	ESTIMATE	LOWER BOUND	UPPER BOUND	CONF. INTERVALS (90 PERCENT)	COVARIANCE MATRIX OF PARAMETER ESTIMATES
MEAN	2	1.0000	2.5000		0.000000
LOG LIKELIHOOD FUNCTION AT MLE =					-280.11

11.087

RECEIVED OF FBI WASH DC

06-09-08 : 00.00

000000000 - 418019

0 2061 0 418010  
01008 0 1 00100-03000

60-2 410018  
BARA-1 4-220700

AMERICAN-DAHL : 7.999  
910817 : 8.09

100-443887-100

CONTACT WITH CONTAMINATED PARADISE  
BRO. AND CO. ST. LOUIS, MO.

NOTE: A SMALL SIGNIFICANCE

(20. 2 . 81) 10014800 1400

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409

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1

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# EXPONENTIAL DENSITY FUNCTION, N=79

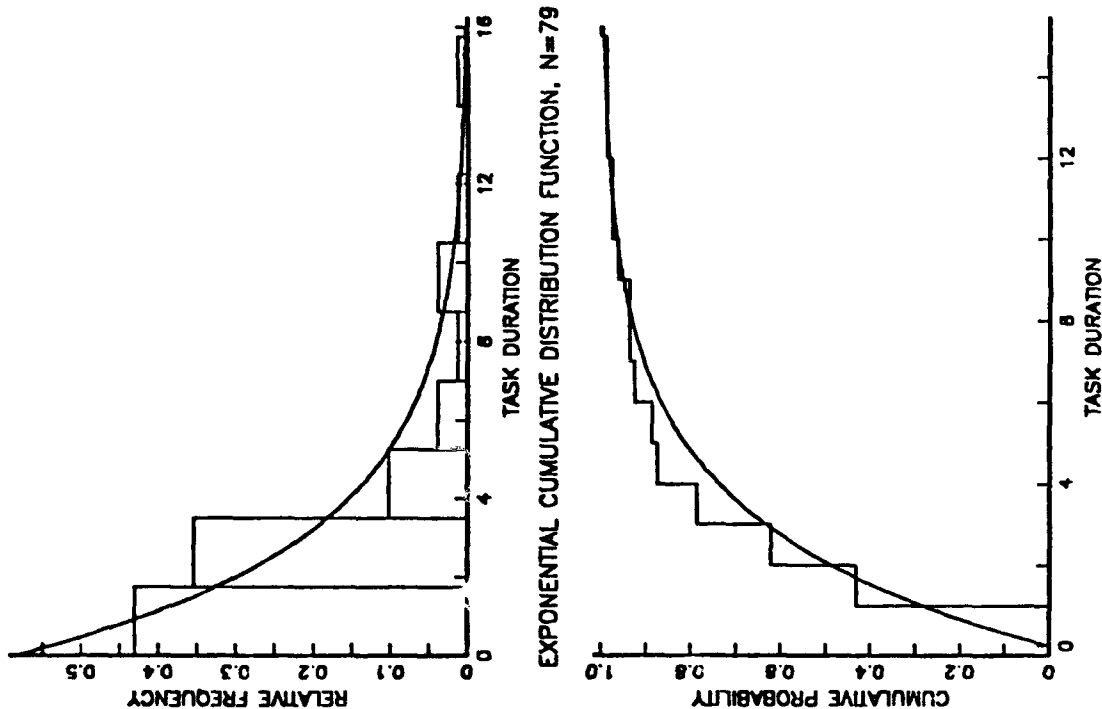


Figure 28. UFF Task Durations [Exp( $\theta=3$ )]

## ANALYSIS OF EXPONENTIAL DISTRIBUTION FIT

DATA : UFF (UPPER,UPPER,UPPER)  
 SELECTION : ALL  
 X AXIS LABEL : TASK DURATION  
 SAMPLE SIZE : 79  
 CENSORING : NONE  
 FREQUENCIES : 1  
 PARAM SPED : SIGMA = 3

COVARIANCE MATRIX OF  
 PARAMETER ESTIMATE  
 NOT AVAILABLE  
 SIGMA : 3

LOG LIKELIHOOD FUNCTION AT MLE = 100.40

MEAN : 8.7348 SAMPLE : FITTED  
 STD DEV : 8.8422 3  
 SKEWNESS : 8.4718 3  
 KURTOSIS : 8.8804 3  
 \* BASED ON MIDPOINTS OF FINITE INTERVALS

PERCENTILES : SAMPLE : FITTED  
 10 : 1.1800 0.1800  
 25 : 1.9100 0.2500  
 50 : 3.6200 0.5000  
 75 : 5.4300 0.7500  
 90 : 7.2400 0.9000  
 95 : 8.0500 0.9500

GOODNESS OF FIT TESTS  
 CHI-SQUARE : 7.00  
 DEG FREED : 6  
 SIGNIF : 0.19418  
 ROLD-SMITH : 0.20267  
 SIGNIF : 0.000001300  
 SHAPIRO-W : 0.95533  
 SIGNIF : 0.405  
 ANDERSON-DARL : 0.4003  
 SIGNIF : 0.101

## CHI-SQUARE GOODNESS OF FIT TABLE

LOWER	UPPER	CHI-SQ	DF	P-VALUE
-INF.	1.70	0.1800	0.000001	0.000001
1.70	3.62	0.2500	0.0100	0.000001
3.62	5.43	0.5000	0.0500	0.000001
5.43	7.24	0.7500	0.1000	0.000001
7.24	8.05	0.9000	0.2000	0.000001
8.05	8.73	0.9500	0.5000	0.000001
8.73	9.50	0.9800	0.7500	0.000001
9.50	10.27	0.9900	1.0000	0.000001
10.27	11.04	0.9950	1.2500	0.000001
11.04	11.81	0.9975	1.5000	0.000001
11.81	12.59	0.9990	1.7500	0.000001
12.59	13.36	0.9995	2.0000	0.000001
13.36	14.13	0.9997	2.2500	0.000001
14.13	14.90	0.9998	2.5000	0.000001
14.90	15.67	0.9999	2.7500	0.000001
15.67	16.44	0.9999	3.0000	0.000001
16.44	17.21	0.9999	3.2500	0.000001
17.21	17.98	0.9999	3.5000	0.000001
17.98	18.75	0.9999	3.7500	0.000001
18.75	19.52	0.9999	4.0000	0.000001
19.52	20.29	0.9999	4.2500	0.000001
20.29	21.06	0.9999	4.5000	0.000001
21.06	21.83	0.9999	4.7500	0.000001
21.83	22.60	0.9999	5.0000	0.000001
22.60	23.37	0.9999	5.2500	0.000001
23.37	24.14	0.9999	5.5000	0.000001
24.14	24.91	0.9999	5.7500	0.000001
24.91	25.68	0.9999	6.0000	0.000001
25.68	26.45	0.9999	6.2500	0.000001
26.45	27.22	0.9999	6.5000	0.000001
27.22	27.99	0.9999	6.7500	0.000001
27.99	28.76	0.9999	7.0000	0.000001
28.76	29.53	0.9999	7.2500	0.000001
29.53	30.30	0.9999	7.5000	0.000001
30.30	31.07	0.9999	7.7500	0.000001
31.07	31.84	0.9999	8.0000	0.000001
31.84	32.61	0.9999	8.2500	0.000001
32.61	33.38	0.9999	8.5000	0.000001
33.38	34.15	0.9999	8.7500	0.000001
34.15	34.92	0.9999	9.0000	0.000001
34.92	35.69	0.9999	9.2500	0.000001
35.69	36.46	0.9999	9.5000	0.000001
36.46	37.23	0.9999	9.7500	0.000001
37.23	38.00	0.9999	10.0000	0.000001
38.00	38.77	0.9999	10.2500	0.000001
38.77	39.54	0.9999	10.5000	0.000001
39.54	40.31	0.9999	10.7500	0.000001
40.31	41.08	0.9999	11.0000	0.000001
41.08	41.85	0.9999	11.2500	0.000001
41.85	42.62	0.9999	11.5000	0.000001
42.62	43.39	0.9999	11.7500	0.000001
43.39	44.16	0.9999	12.0000	0.000001
44.16	44.93	0.9999	12.2500	0.000001
44.93	45.70	0.9999	12.5000	0.000001
45.70	46.47	0.9999	12.7500	0.000001
46.47	47.24	0.9999	13.0000	0.000001
47.24	48.01	0.9999	13.2500	0.000001
48.01	48.78	0.9999	13.5000	0.000001
48.78	49.55	0.9999	13.7500	0.000001
49.55	50.32	0.9999	14.0000	0.000001
50.32	51.09	0.9999	14.2500	0.000001
51.09	51.86	0.9999	14.5000	0.000001
51.86	52.63	0.9999	14.7500	0.000001
52.63	53.40	0.9999	15.0000	0.000001
53.40	54.17	0.9999	15.2500	0.000001
54.17	54.94	0.9999	15.5000	0.000001
54.94	55.71	0.9999	15.7500	0.000001
55.71	56.48	0.9999	16.0000	0.000001
56.48	57.25	0.9999	16.2500	0.000001
57.25	58.02	0.9999	16.5000	0.000001
58.02	58.79	0.9999	16.7500	0.000001
58.79	59.56	0.9999	17.0000	0.000001
59.56	60.33	0.9999	17.2500	0.000001
60.33	61.10	0.9999	17.5000	0.000001
61.10	61.87	0.9999	17.7500	0.000001
61.87	62.64	0.9999	18.0000	0.000001
62.64	63.41	0.9999	18.2500	0.000001
63.41	64.18	0.9999	18.5000	0.000001
64.18	64.95	0.9999	18.7500	0.000001
64.95	65.72	0.9999	19.0000	0.000001
65.72	66.49	0.9999	19.2500	0.000001
66.49	67.26	0.9999	19.5000	0.000001
67.26	68.03	0.9999	19.7500	0.000001
68.03	68.80	0.9999	20.0000	0.000001
68.80	69.57	0.9999	20.2500	0.000001
69.57	70.34	0.9999	20.5000	0.000001
70.34	71.11	0.9999	20.7500	0.000001
71.11	71.88	0.9999	21.0000	0.000001
71.88	72.65	0.9999	21.2500	0.000001
72.65	73.42	0.9999	21.5000	0.000001
73.42	74.19	0.9999	21.7500	0.000001
74.19	74.96	0.9999	22.0000	0.000001
74.96	75.73	0.9999	22.2500	0.000001
75.73	76.50	0.9999	22.5000	0.000001
76.50	77.27	0.9999	22.7500	0.000001
77.27	78.04	0.9999	23.0000	0.000001
78.04	78.81	0.9999	23.2500	0.000001
78.81	79.58	0.9999	23.5000	0.000001
79.58	80.35	0.9999	23.7500	0.000001
80.35	81.12	0.9999	24.0000	0.000001
81.12	81.89	0.9999	24.2500	0.000001
81.89	82.66	0.9999	24.5000	0.000001
82.66	83.43	0.9999	24.7500	0.000001
83.43	84.20	0.9999	25.0000	0.000001
84.20	84.97	0.9999	25.2500	0.000001
84.97	85.74	0.9999	25.5000	0.000001
85.74	86.51	0.9999	25.7500	0.000001
86.51	87.28	0.9999	26.0000	0.000001
87.28	88.05	0.9999	26.2500	0.000001
88.05	88.82	0.9999	26.5000	0.000001
88.82	89.59	0.9999	26.7500	0.000001
89.59	90.36	0.9999	27.0000	0.000001
90.36	91.13	0.9999	27.2500	0.000001
91.13	91.90	0.9999	27.5000	0.000001
91.90	92.67	0.9999	27.7500	0.000001
92.67	93.44	0.9999	28.0000	0.000001
93.44	94.21	0.9999	28.2500	0.000001
94.21	94.98	0.9999	28.5000	0.000001
94.98	95.75	0.9999	28.7500	0.000001
95.75	96.52	0.9999	29.0000	0.000001
96.52	97.29	0.9999	29.2500	0.000001
97.29	98.06	0.9999	29.5000	0.000001
98.06	98.83	0.9999	29.7500	0.000001
98.83	99.60	0.9999	30.0000	0.000001
99.60	100.37	0.9999	30.2500	0.000001
100.37	101.14	0.9999	30.5000	0.000001
101.14	101.91	0.9999	30.7500	0.000001
101.91	102.68	0.9999	31.0000	0.000001
102.68	103.45	0.9999	31.2500	0.000001
103.45	104.22	0.9999	31.5000	0.000001
104.22	104.99	0.9999	31.7500	0.000001
104.99	105.76	0.9999	32.0000	0.000001
105.76	106.53	0.9999	32.2500	0.000001
106.53	107.30	0.9999	32.5000	0.000001
107.30	108.07	0.9999	32.7500	0.000001
108.07	108.84	0.9999	33.0000	0.000001
108.84	109.61	0.9999	33.2500	0.000001
109.61	110.38	0.9999	33.5000	0.000001
110.38	111.15	0.9999	33.7500	0.000001
111.15	111.92	0.9999	34.0000	0.000001
111.92	112.69	0.9999	34.2500	0.000001
112.69	113.46	0.9999	34.5000	0.000001
113.46	114.23	0.9999	34.7500	0.000001
114.23	115.00	0.9999	35.0000	0.000001
115.00	115.77	0.9999	35.2500	0.000001
115.77	116.54	0.9999	35.5000	0.000001
116.54	117.31	0.9999	35.7500	0.000001
117.31	118.08	0.9999	36.0000	0.000001
118.08	118.85	0.9999	36.2500	0.000001
118.85	119.62	0.9999	36.5000	0.000001
119.62	120.39	0.9999	36.7500	0.000001
120.39	121.16	0.9999	37.0000	0.000001
121.16	121.93	0.9999	37.2500	0.000001
121.93	122.70	0.9999	37.5000	0.000001
122.70	123.47	0.9999	37.7500	0.000001
123.47	124.24	0.9999	38.0000	0.000001
124.24	125.01	0.9999	38.2500	0.000001
125.01	125.78	0.9999	38.5000	0.000001
125.78	126.55	0.9999	38.7500	0.000001
126.55	127.32	0.9999	39.0000	0.000001
127.32	128.09	0.9999	39.2500	0.000001
128.09	128.86	0.9999	39.5000	0.000001
128.86	129.63	0.9999	39.7500	0.00000

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